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# An Options-Based Solution to the Sequential Auction Problem<sup>1</sup>

Adam I. Juda<sup>2</sup> and David C. Parkes<sup>3</sup>

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## Abstract

The *sequential auction problem* is commonplace in open, electronic marketplaces such as eBay. This problem occurs when a buyer faces a complex strategic problem in bidding across multiple auctions, each selling similar or essentially identical items and when the buyer would have a simple, truth-revealing strategy if there was but a single auction event. Our model allows for multiple, distinct goods and market dynamics with buyers and sellers that arrive over time. Sellers each bring a single unit of a good to the market while buyers can have values on bundles of goods. We model each individual auction as a second-price (Vickrey) auction and propose an *options-based, proxied* solution to provide price and winner-determination coordination across auctions. While still allowing for temporally uncoordinated market participation, this options-based approach solves the sequential auction problem and provides a market in which truthful bidding is a weakly dominant strategy for buyers. An empirical study suggests that this coordination can enable a significant efficiency and revenue improvement over the current eBay market design, and highlights the various effects on performance of complex buyer valuations (buyers with *substitutes* and *complements* valuations) and in varying the market liquidity.

*Key words:* Coordination problems, Preferences, Dynamic auctions, Proxy agents, Options, Strategyproofness, Electronic markets.

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## 1 Introduction

Electronic markets generate significant new trading opportunities and expand the opportunity for the dynamic pricing of goods, and lead to improved market efficiency in many settings [9,10,54]. Electronic markets find application not only for person-to-person transactions (e.g., auctions), but also increasingly for business-to-consumer auctions such as selling surplus inventory [35] and business-to-business sourcing events [50].

But despite the new efficiencies offered by electronic markets, for example by enabling the application of optimization to decision making in many parts of the supply chain, the role of *automated trading agents* – although long envisioned by artificial intelligence researchers [51,23,1] – remains more fiction than reality. (One notable exception is the significant role of automated trading for financial securities [20].) One major impediment to the adoption of automated trading agents is that users often have insufficient trust of software agents that would work on their behalf [14,33]. Users may even have higher expectations for automated agents than human agents in regard to what constitutes acceptable behavior [53]. For example, the proposal by the London International Financial Futures Exchange (Liffe) to introduce automated trading was the cause of much debate.<sup>4</sup> A key area of concern was that users could not be certain that the automated systems would behave “optimally” in all situations.

One important, if unsurprising, aspect that seems important in the adoption of automated agents is that these agents avoid, and be seen to avoid, making mistakes [41]. In the context of electronic markets, an insight offered by *mechanism design*, the subdiscipline of microeconomics that seeks to design protocols to achieve system-wide objectives with self-interested agents, is that one can sometimes design protocols that *simplify* the strategic considerations of bidding agents. Classic solutions, such as those offered by the Vickrey-Clarke-Groves (VCG) mechanism [30], provide this simplicity via the property of *strategyproofness* (which brings truthful bidding into a dominant strategy equilibrium), and in addition provide market efficiency. But we argue that they are often not applicable in practice because they require too much temporal coordination on the part of participants. Electronic markets such as eBay<sup>5</sup> allow a would-be seller to decide when to sell goods in the marketplace, and buyers can visit the marketplace at times of their own choosing. The VCG mechanism on the other hand requires that buyers and sellers be grouped into

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<sup>4</sup> The Financial Times said at the time that “*Electronic trading is the biggest single issue to face the futures community today and the industry has long confronted a philosophical split on its merits*”, in an article “Liffe’s new automated trading system has sparked a debate on automated trading,” November 30, 1989.

<sup>5</sup> [www.ebay.com](http://www.ebay.com)

a single, coordinated auction with all bids placed at the same time and all goods sold at the same time.<sup>6</sup>

In fact, the individual auctions on eBay are very similar to *single-item* second-price (Vickrey) auctions rather than the more general VCG mechanism. The most significant difference is that eBay provides, via the use of a mandatory proxy agent that bids on behalf of a buyer, a “staged Vickrey auction” such that a buyer can effectively increase her bid price at any time until an auction closes.<sup>7</sup> Electronic markets such as eBay do not provide a VCG mechanism at the level of a category of goods, for instance all LCD monitors in the market, arguably because this would require too much temporal coordination on the part of buyers and sellers. A second issue would be determining how to define the scope for such an event to include a suitable domain of goods likely to subsume most of those of interest to a set of potential buyers. A third issue is that at some point the computation and communication cost for running such a large, coordinated mechanism would also get prohibitive given that winner-determination for combinatorial auctions is **NP**-hard [49].

But the absence of such a large-scale coordination mechanism in markets such as eBay leads to strategic complexity for participants. Despite a (weak) strategic equivalence between an individual auction on eBay and the Vickrey auction [36,55], there are many reasons for buyers not to truthfully bid their value in any one auction. One reason follows from the auction being staged rather than sealed-bid. Because some bidders may be “followers” – bidding up when others do – it can be rational to delay until the last minute and “snipe” to avoid driving up competitors’ bid prices [40,3]. The *sequential auction problem* provides another reason. This relates to issues that arise when composing strategies across a sequence of auctions.

For an example, suppose that multiple copies of essentially identical items are offered for sale sequentially. For example, Alice may want an LCD monitor, and could potentially bid in either a 1 o’clock or 3 o’clock auction. Alice would prefer to participate in the auction that will have the lower winning price, but she cannot determine beforehand which auction that will be. As a result, she could end up winning in the “wrong” auction, that is the auction

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<sup>6</sup> Generalizations of the VCG mechanism to dynamic settings have been developed [43], but are not immediately applicable in this context for reasons discussed in the related work.

<sup>7</sup> While an eBay auction is open, a buyer provides her automated proxy agent with a bid ceiling. While the ceiling the agent has received is greater than the winning price and the agent is not winning, the agent will submit a bid some amount,  $\epsilon$ , above the current winning price (where  $\epsilon > 0$  is set by eBay anywhere from cents to dollars depending on the value of the item). Therefore, when an auction ends, the winning buyer will pay a price  $\epsilon$  above the highest ceiling another buyer submitted, and the outcome is nearly identical to the outcome of the Vickrey auction.

with the higher price. A related example of the sequential auction problem is familiar from the *exposure problem* studied in *simultaneous* ascending price auctions [12], which also exists in our setting when a buyer desires a bundle of goods but must participate in auctions on individual items. For example, if Alice values a video game console by itself for \$200, a video game by itself for \$30, and both a console and game for \$250, she must determine how much of the \$20 of synergy value to include in her bid when bidding for the console alone. If Alice incorporates some of the synergy value (e.g., by placing a bid of \$210), she may incur a loss if she can not subsequently win the video game for less than \$40.<sup>8</sup>

The main technical question addressed in this paper is: *can one design an efficient marketplace for temporally uncoordinated buyers and sellers, and distinct goods, in which buyers have a simple, dominant-strategy bidding strategy?* As a solution we propose a *real-options based* market infrastructure, coupled with *proxy bidding*, that enables simple, yet optimal, bidding strategies while retaining the dynamic arrivals and departures that are a defining feature of electronic markets such as eBay. Our main assumptions are that:

- Buyers may have general valuations on bundles of distinct goods, but are interested in at most one unit of each of these goods.
- Each buyer has an arrival time and a departure time in the market, and is indifferent between buying items at any time before her departure and with zero value after her departure.
- Each seller offers a single unit of a good, has no intrinsic value for the good, and is willing to wait until the departure time of a winning buyer to receive her payment.
- Sellers are non-strategic and truthfully report arrival and departure to the market, which defines the interval of time during which they are willing to sell the good.
- Goods can be placed into equivalence classes so that all buyers are indifferent between goods in the same class.

Note that we will not need to assume that buyers can only participate under one identity or prevent buyers from re-entering the market; this provides robustness to the false-name bidding considered by Yokoo [59].

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<sup>8</sup> A third reason for strategic complexity in markets such as eBay can be that the quality of items may be uncertain and buyers may adjust their belief about the value of the item based on others' bids; this is the so-called *interdependent* values model of auction theory. We do not consider interdependent value domains in this paper. This makes our results applicable instead to markets in which buyers know their value for goods and can determine this without seeing the bids from others. An example of such a domain is provided by our empirical study on the eBay market for a Dell LCD monitor.

The final assumption is probably the strongest. But we note that many items listed in an eBay auction are essentially identical to those in other auctions, and especially in categories such as Consumer Electronics, where the sum of all successfully closed listings during 2005 was U.S. \$3.5B (of U.S. \$44B in total for all of eBay) [22]. This category is the focus of our empirical analysis, in which we consider auctions for 19" Dell LCD monitors (Model E193FP) conducted on eBay during the summer of 2005. Moreover, we do not require that an individual buyer necessarily has a different value for goods in different classes, and different buyers need not hold the same value for goods within a particular class. The only property that is important is that every buyer's individual value is the same for any good within a class.<sup>9</sup>

In the options-based solution, a seller auctions an option for her good rather than auctioning the good directly. The option will ultimately either lead to a sale or require the seller to return to the market and offer another option on the same good. By participation in the framework, sellers agree to allow proxy agents to *price-match* their goods against others of equal type, with the payment a seller finally receives defined in terms of the *minimal price that the winning bidder could have bid and still traded with some seller in some auction during her arrival-departure interval*. As noted above, we assume that sellers have no intrinsic value for the good and moreover are non-strategic in that they will truthfully report their temporal constraints in the marketplace.

All buyers in our framework must interact through proxy agents and do so by reporting a value on all possible bundles of goods of interest along with a departure time. While such an enumeration may seem daunting at first glance, there are several reasons not to view this as a major concern in consumer markets. First, a very common purchasing scenario is for a buyer to want a single item or to be indifferent among only a few different items. Second, Cantillon and Pesendorfer [13] and Sandholm [50] provide empirical support that buyers can manage to construct bids in large combinatorial settings. Third, a number of expressive, concise bidding languages have been developed for combinatorial auctions [15,39,7].

A proxy agent uses the reported information about value and departure time to determine how to bid for options and also to determine which options held at the buyer's reported departure time to exercise. The options that maximize the buyer's surplus given the reported valuation are exercised and all other options are returned to the seller. The options-based protocol is useful because it makes *truthful and immediate revelation to a proxy a dominant strategy for*

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<sup>9</sup> In the absence of a third party logistics partner, such as Amazon, that offers fulfillment and commits to the quality of a good (e.g. new, and "in box") it is likely that sellers could improve revenue in the short-term by overstating the quality of their item and misleading buyers in the marketplace. However, and just as on eBay, a well-functioning reputation system should mitigate this concern [47].

*buyers, whatever the future auction dynamics.* Thus it can be seen as a method to generalize eBay’s existing proxy scheme to handle the sequential auction problem in suitable categories of goods, while extending to embrace dynamic, combinatorial auctions.

An empirical analysis is performed using data on eBay auctions for 19” Dell LCD monitors (Model E193FP) sold from 27 May, 2005 through 1 October, 2005. A conservative estimate is that an improvement in efficiency and revenue of around 4% and 9% respectively would be enabled through an options-based scheme. This estimate is generated on the basis of non-parametric estimation of the true value of buyers for items, generalizing a method due to Haile and Tamer [24] to sequential auctions. The eBay analysis also informs an extensive set of simulation experiments, in which we explore the effect of *substitutes* (“I want  $A$  or  $B$ ”) and *complements* (“I only want  $A$  if I also get  $B$ ”) valuations on the efficiency of the options-based scheme and also consider the impact of market liquidity. Buyer populations with substitutes valuations can hamper the efficiency of the marketplace because of hold-up problems in which a buyer’s proxy holds a number of options that ultimately go unexercised but were unavailable to other buyers. However we find that this effect is mitigated when individual buyers have negatively correlated values across items. We also find that the buyer-to-seller ratio (a measure of *liquidity*) plays a critical role in market efficiency in the context of substitutes valuations. Efficiency first decreases and then increases as the buyer-to-seller ratio increases and the market becomes more competitive. For a low buyer-to-seller ratio the market remains efficient with substitutes valuations because there is plenty of supply. Market efficiency is also high for relatively large buyer-to-seller ratios (above 4:1 in our simulations) and substitutes valuations because increased competition segments the market; buyers tend to be competitive on only a small number of goods. In the context of *complements* valuations we find that market efficiency is fairly insensitive to positive or negative correlation in value across items and remains reasonably high.

By providing a system in which buyers possess a simple, dominant strategy, the proxy-based solution arguably reduces the participation costs of buyers. While the impact of this improvement is hard to estimate, this can be expected to both improve buyer loyalty to sellers and also make the market more appealing for new entrants. These two effects ought to preserve and enhance the health of the market and maintain seller revenue in the long term.

There are a variety of extensions beyond the immediate purview of this work which could further enhance the performance of the options-based scheme. We mention some of these opportunities, and challenges, in discussion and in offering conclusions. In outline, we will first discuss related work, and then use Section 2 to introduce the model and define and characterize the sequential auction problem. Section 3 describes the options-based scheme, giving

examples and a complexity analysis. Section 4 provides a strategic analysis and a worst-case efficiency analysis. Section 5 presents an experimental study, first on eBay data (the LCD market) and then extending – in simulation – to consider substitutes and complements preferences and the effect of market liquidity. Section 6 discusses the challenges of allowing proxy agents to bid less aggressively, thus mitigating hold-up. Section 7 concludes.

## 1.1 Related Work

A number of authors have analyzed sequential auctions selling the same item with buyers interested in buying just a single item. This “multiple copies problem” is often studied in the context of explaining sniping behavior; see also Ockenfels and Roth [40], who give a collusion-based explanation for sniping. From the perspective of developing models for the multiple copies problem, Stryzowska [56] models the problem as one of a dynamic multi-unit auction, allowing for explanations of sniping as well as for bidding multiple times within an auction. Hendricks et al. [27] demonstrate that sniping is a symmetric equilibrium in the absence of a “BuyItNow” opportunity (wherein a buyer can choose to buy an item at any point for some fixed price). Wang [57] demonstrates, using a two-period model, how sniping in the first period is a unique equilibrium and Zeithammer [60] provides an equilibrium model for strategic sellers and forward-looking buyers. Notably, none of this prior work considers buyers that are able to participate in more than two auctions, while we consider settings in which buyers may participate in an arbitrary number of auctions. Peters and Severinov [45] also allow this more general capability and characterize a perfect Bayesian equilibrium where sellers set a reserve price equal to their true costs. These authors consider neither buyers entering at random times nor auctions closing at different times, both of which are addressed in our work. While these papers provide a Bayesian-Nash analysis of models that approximate current eBay-like markets, we study the dominant strategy equilibrium in an options-based variation on current markets.

Problems of the same kind as the sequential auction problem were previously observed by Wellman and Wurman [58] in the context of boundaries between multiple mechanisms, and later discussed by Parkes [42] and Ng et al. [37,38] in the context of “strategyproof computing.” The problem has often been identified in the context of simultaneous ascending price auctions, where it is termed the *exposure problem* [12]. Previous work to address the exposure problem has considered two different directions. First, one can change the mechanism and define an expressive bidding language and a strategyproof mechanism, as seen in work on combinatorial auctions [49]. Second, one can attempt to provide automated bidding agents with sophisticated strategies, as seen for example in the work of Boutilier et al. [6], Byde et al. [11], Anthony



and Jennings [1], Reeves et al. [46], and Gerding et al. [21]. Unfortunately, it seems hard to design artificial agents with equilibrium bidding strategies, even for a simultaneous ascending price auction (i.e. without dynamic arrivals of new sellers) and all these papers make significant assumptions.

Iwasaki et al. [29] have previously considered the use of options in the context of a single, monolithic, auction design to help buyers with marginal-increasing values avoid exposure in a multi-unit, homogeneous item auction. Sandholm and Lesser [52] have considered options in the form of *leveled commitment contracts* for facilitating multi-way recontracting in a completely decentralized market place. Rothkopf and Engelbrecht-Wiggans [48] discuss the advantages associated with the use of options for selling coal mine leases. To the best of our knowledge, ours is the first work to study the role of options as a method to enable dominant strategies in the context of dynamic auctions. Gopal et al. [22] have considered the use of options for reducing exposure to risk in the context of the sequential auction problem. Our work differs in a number of ways, including how the options are priced, which buyers obtain options, and in how much risk remains with buyers once options are used. Buyers still face risk and have no dominant strategy in the method of Gopal et al. [22].

The technical contribution of this paper is related to *online mechanism design* [32,44,26,43]. In online mechanism design (online MD), one seeks an incentive mechanism for a dynamic environment in which agents arrive and depart and in which there is uncertainty about the future. In the analysis of our protocol, we slightly generalize the price-based characterization of Hajiaghayi et al. [25] to establish a dominant strategy equilibrium for buyers, creating a truthful online combinatorial auction from an uncoordinated sequence of single item auctions. The options-based scheme generalizes the earlier protocol in Hajiaghayi et al. [25] to combinatorial settings but is equivalent in an environment in which everyone is buying and selling a single unit of the same kind of good, albeit with a decentralized architecture. Dynamic VCG mechanisms [4,44,16] are unsuitable because they handle only one-sided markets, require *optimal* allocation policies (which may be intractable in domains of interest), and provide a Bayesian-Nash equilibrium rather than full strategyproofness. A dynamic extension of the expected externality mechanism [2,18] is suitable for dynamic two-sided markets, but would again require an optimal policy and provide only a weak guarantee about the relationship between a buyer's payment and her bid price. Bredin et al. [8] provide simple two-sided dynamic auction protocols that support dominant-strategy equilibrium, but only for an environment in which all buyers and sellers are interested in trading a single unit of a commodity good.

It is interesting to note that some of the strategic difficulties that buyers face in uncoordinated electronic auctions such as eBay are also faced by consumers acquiring items in the retail sector and it is interesting that retail stores have

developed policies that can be interpreted as assisting customers in this regard. *Return policies* alleviate the exposure problem by allowing customers to return goods at the purchase price while *price matching* alleviates the multiple copies problem by allowing buyers to receive from sellers *after purchase* the difference between the price paid for a good and a lower price found elsewhere [17,34]. A concern that is discussed in the academic literature in regard to these practices in the retail sector is that they can be *anti-competitive*, with sellers using them as a commitment device for avoiding price competition [28]. We do not foresee this issue in the context of proxied, sequential auctions as proposed in this paper because the prices are not set by sellers but rather determined by competition on the buy side.

## 2 Preliminaries: The Sequential Auction Problem

In this section we introduce the formal model and define and characterize the sequential auction problem which motivates our work.

### 2.1 The Model

In our domain, there are  $K$  different kinds of goods (often referred to as *items*), denoted with set  $G$  and  $G_1, \dots, G_K$  for the individual kinds of goods, a set  $B$  of buyers (perhaps unbounded), a set  $S$  of sellers (perhaps unbounded), and  $T = \{0, 1, \dots\}$  discrete time periods. Each buyer  $i \in B$  has a utility function parameterized with *type*  $\theta_i = (a_i, d_i, v_i) \in \Theta$ , where  $\Theta$  is the set of all possible types, defining her *arrival time*  $a_i \in T$ , *departure time*  $d_i \geq a_i \in T$ , and *valuation*  $v_i(L) \geq 0$  for every possible bundle of goods  $L \subseteq G$ . We assume that no buyer demands more than a single unit of each good and write  $L_k \in \{0, 1\}$  to denote whether or not bundle  $L$  contains a unit of good  $G_k$ . We assume *free disposal* and *normalization*, with  $v_i(L) \geq v_i(S)$  for  $L \supseteq S$  and  $v_i(\emptyset) = 0$ .

The semantics of arrival and departure are such that buyer  $i$  has value  $v_i(L) \geq 0$  for a bundle of goods  $L \subseteq G$  that is allocated (potentially in multiple pieces) across periods  $[a_i, \dots, d_i]$  and is indifferent to goods allocated outside of this time interval; i.e., buyers have no value for goods allocated after departure  $d_i$ . Buyers have *quasi-linear* utilities, so that the utility of buyer  $i$  for receiving bundle  $L$  and paying  $p \in \mathbb{R}_{\geq 0}$ , in some period no later than  $d_i$ , is  $u_i(L, p) = v_i(L) - p$ . For the sake of analysis it is convenient to assume the existence of a *maximal patience*, such that  $d_i - a_i \leq \Delta_{\max}$ , for some constant  $\Delta_{\max}$ .

The arrival time models the period in which a buyer first realizes her demand and enters the market. The departure time models the period in which a buyer

loses interest in acquiring the good(s) from this marketplace. For example, a buyer may lose interest in items whose value is realized at a specific past moment in time (e.g., Saturday night movie tickets), or because she simply wishes to take advantage of an outside opportunity to acquire the item (e.g., a buyer deciding to acquire an item at a posted price on a certain date). Modeling a buyer's value as constant during the arrival-departure interval is especially reasonable when the interval is small in relation to the time over which the good(s) will be used; e.g., an LCD monitor that a buyer plans on using for 3 years provides a buyer with roughly equivalent value if held for 1000 days or 998 days.

Each seller  $j \in S$  brings a single unit of one kind of good, denoted  $k_j \in G$  to the market and is assumed to have no intrinsic value for the good. Seller  $j$  has an *arrival time*,  $a_j \in T$ , which models the period in which she is first interested in listing the item, and a *departure time*,  $d_j \geq a_j \in T$ , which models the latest period in which she is willing to consider having an auction for the item close. By listing a good for sale until  $d_j$ , a seller is indicating her willingness to receive payment by the end of the reported departure of the winning buyer.<sup>10</sup> We will not be concerned with strategic behavior by sellers and focus our analysis on removing the sequential auction problem for buyers whatever the strategy of sellers.

The options-based framework provides a *direct-revelation* (online) mechanism in which each buyer  $i$  interacts with the market only once by declaring a *bid*,  $b_i \in \Theta$ , which is a (perhaps untruthful) claim about her type. Denoting the bids from all buyers as  $b = (b_1, \dots, b_n)$ , a direct-revelation mechanism determines an *allocation*  $x_i(b) \subseteq G$  and *payment*,  $p_i(b) \geq 0$ , to each buyer. The outcome also depends on the sell-side, i.e. the goods that are brought to market, but we leave this dependence silent in the notation. Since this is an *online* setting, with the bids are reported over time, the allocation and payment functions must be *online computable*; i.e. if a buyer is to be allocated an item for sale in period  $t$  then this must be known based on information available up to and including period  $t$ .

In this dynamic environment we follow earlier models [32,26,25], and assume *limited misreports*, so that a buyer is assumed to be unable to bid before her true arrival period. Formally, we adopt  $Y(\theta_i) \subsetneq \Theta$  to denote the set of available misreports, such that  $\theta'_i = (a'_i, d'_i, v'_i) \in Y(\theta_i) \Rightarrow a'_i \geq a_i$ . Given these limited misreports, a *dominant strategy equilibrium* requires that every buyer,  $i \in B$ ,

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<sup>10</sup> This delay in receiving payment is not significantly different than what sellers on eBay currently endure in practice. An auction on eBay closes at a specific time, but a seller must wait until a buyer relinquishes payment before being able to realize the revenue, an amount of time that could easily be days (if payment is via a money order sent through courier) to much longer (if a buyer is slow but not overtly delinquent in remitting her payment).

has a bidding strategy  $b_i^*(\theta_i) \in Y(\theta_i) \subsetneq \Theta$  that maximizes her utility whatever the bids of other buyers and for all possible future dynamics. Formally, this requires that:

$$v_i(x_i(b_i^*(\theta_i), b_{-i})) - p_i(b_i^*(\theta_i), b_{-i}) \geq v_i(x_i(b'_i, b_{-i})) - p_i(b'_i, b_{-i}), \quad (2.1)$$

$$\forall b'_i \in Y(\theta_i), \forall b_{-i} \in \Theta_{-i}, \forall \theta_i \in \Theta,$$

for every buyer  $i \in B$ , where  $b_{-i} = (b_1, \dots, b_{i-1}, b_{i+1}, \dots, b_n)$ , and  $\Theta_{-i} = \Pi_{\neq i} \Theta$ , i.e. the joint type space of the other buyers. A mechanism is said to be *strategyproof* when reporting the *true* type is a dominant strategy equilibrium. Note that this holds for all dynamics and thus for all sell-side strategies.

Because we are concerned with markets in which there may be multiple individual auctions available to a buyer, it is useful to adopt the terminology *locally strategyproof* for an auction in which truthful bidding is a dominant strategy *if a buyer was restricted to bidding in that one auction* [42]. This would be true, for example, if the auction is for a unique item that is only available for auction once in a buyer's lifetime (e.g., a piece of artwork for which there is no substitute), or if a buyer is very impatient. In Section 5 we present empirical evidence to suggest that this is generally not true in markets such as eBay.

## 2.2 The Sequential Auction Problem

The sequential auction problem describes the strategic problem that can face a buyer even though she faces a sequence of locally strategyproof auctions. Consider the following two motivating examples:

**Example 1** *Alice values acquiring one ton of sand before Wednesday for \$1,000. Bob will hold a Vickrey auction for one ton of sand on Monday, and another such auction on Tuesday. Alice has no dominant bidding strategy because she cannot predict whether the price of the Tuesday auction will be greater or less than the price of the Monday auction, and she needs to know this price when deciding on an optimal bidding strategy on Monday.*

**Example 2** *Alice values one ton of sand with one ton of stone at \$2,000. Bob holds a Vickrey auction for one ton of sand on Monday. Charlie holds a Vickrey auction for one ton of stone on Tuesday. Alice has no dominant bidding strategy because she needs to know the price for stone on Tuesday in order to know how much to bid for sand on Monday. If Alice bids too high on Monday, she may be left with one ton of sand but no ability to buy the one ton of stone required to complete her construction project. If Alice bids too low on Monday, she might forfeit the opportunity to buy both the sand and stone, for*

example if the price of stone on Tuesday is low.

**Definition 1 (sequential auction problem)** *The sequential auction problem exists when a buyer has no dominant bidding strategy in a sequence of auctions, despite each auction being locally strategyproof.*

In characterizing the sequential auction problem, we consider a sequence of auctions,  $\mathcal{A}$ , that occur in periods  $t \in [a_i, \dots, d_i]$ . There may be uncertainty about this sequence of auctions from the perspective of the buyer. Let  $\mathcal{P}$ , with  $\vec{p} \in \mathcal{P}$ , denote the set of possible prices that the buyer may face, where a particular realization of prices corresponds to a particular realization of supply (i.e., auctions  $\mathcal{A}$ ) and demand. A bundle  $L \subseteq G$  is *strictly valued* to buyer  $i$  with valuation  $v_i$ , if her value  $v_i(L) > \max_{Q \subsetneq L} v_i(Q)$ . The empty bundle is strictly-valued. Consider  $\vec{p} \in \mathcal{P}$  and denote by  $IB_i(\vec{p}) \in \{0, 1\}^m$ , where  $m = |\vec{p}|$ , an *optimal buying schedule* given prices  $\vec{p}$ , i.e. maximizing  $v_i(S) - IB_i(\vec{p}) \cdot \vec{p}$ , where  $S$  is the bundle purchased and  $S$  is restricted to a strictly-valued bundle. Let  $\vec{p}^{\prec k}$ , for  $k \geq 1$ , denote the first  $k$  elements of  $\vec{p}$ .

Fix a particular optimal buying schedule  $IB_i$ . An *interesting good* is one that is purchased for some prices  $\vec{p} \in \mathcal{P}$ . An auction is interesting if it sells a good that should be purchased in that auction for some prices  $\vec{p} \in \mathcal{P}$ .

We say that (a) *all interesting goods are uniquely supplied* and (b) *all uniquely supplied goods have a certain marginal value* if and only if there exists some optimal buying schedule,  $IB_i$ , for which:

(a) whenever good  $G_k$  is purchased in the auction immediately following initial prices,  $\vec{p}^{\prec k}$ , there are no prices  $\vec{p}, \vec{q} \in \mathcal{P}$  with  $\vec{p}^{\prec k} = \vec{q}^{\prec k}$  such that the buyer should buy  $G_k$  in a later auction (“good  $G_k$  is uniquely supplied”).

(b) for every (uniquely supplied) good  $G_k$ , then for all  $\vec{p}, \vec{q} \in \mathcal{P}$  s.t.  $\vec{p}^{\prec k} = \vec{q}^{\prec k}$ , and  $\vec{p}^k \geq \vec{q}^k$ , where  $k$  is the index of the auction for good  $G^k$  and  $\vec{p}^k$  and  $\vec{q}^k$  denote the price in that auction, then  $G_k \in IB_i(\vec{p}) \Rightarrow G_k \in IB_i(\vec{q})$  (“good  $G_k$  has a certain marginal value”).

In particular, if there is no such buying schedule then there must be either an interesting good that is not uniquely supplied ( $\neg(a)$ ) or an interesting good that is uniquely supplied but has an uncertain marginal value ( $\neg(b)$ ). Note that if a good  $G_k$  is only ever available in just one auction, and this auction is always the last auction faced by a buyer, then this good must have a certain marginal value because there are no future auctions about which there can be uncertainty. *As a special case, when a buyer faces only a single auction then the market must satisfy both property (a) and (b).* It is also easy to see that if a buyer’s valuation function is induced from a linear-additive value on each good that she acquires then (b) will trivially hold, with  $G_k \in IB_i(\vec{p})$  if and only if  $\vec{p}^k \leq v^k$ , the buyer’s value for good  $G_k$ .

**Proposition 1** *Given a sequence of locally strategyproof single-item auctions, the sequential auction problem exists if and only if at least one of the following three conditions is true:*

- ( $\neg(a)$ ) there is an interesting good that is not uniquely supplied, or*
- ( $\neg(b)$ ) there is a uniquely supplied good with an uncertain marginal value, or*
- (c) there is more than one interesting auction and competitors' bids can be conditioned on the buyer's past bids.*

**Proof.** ( $\Rightarrow$ ) Assume the sequential auction problem (SAP) and  $(a) \wedge (b) \wedge \neg(c)$  for a contradiction. First, by  $\neg(c)$  the bids of other buyers are fixed and independent of the bids of buyer  $i$  and therefore the prices in each auction in  $\mathcal{A}$  are independent of the strategy of the buyer (since the auctions are also locally strategyproof). Given this, by (a), there is an optimal buying schedule  $IB_i$  for which each good, when purchased in a particular auction, should never be purchased in any other auction. Focus on the auctions in which a good may be purchased and assume, without loss of generality, that these interesting goods are sequenced from  $G_1$  to some good  $G_m$ . We establish a dominant bidding strategy by induction on the number of the auction. The induction hypothesis is that the agent holds the optimal bundle of goods at the start of every auction, i.e. the bundle that it should have purchased given knowledge of eventual prices  $\vec{p}$ . *Base case:* trivial because no goods have been offered and the optimal bundle is empty. *Inductive case:* by (b) there is some value  $v^k$  (perhaps  $\infty$  or 0) s.t. the good should be purchased if and only if  $\vec{p}^k \leq v^k$  and whatever the prices in future auctions. Whether or not the good is acquired, the buyer holds the optimal bundle for  $\vec{p}$  since it held the optimal bundle at the start of this auction and it has made an optimal decision. This is a contradiction with the SAP because we have constructed a buying schedule that can be implemented online.<sup>11</sup>

( $\Leftarrow$ ) Show that  $\neg(a) \vee \neg(b) \vee (c)$  implies the SAP.  $\neg(a)$  implies the SAP (and even if  $(b) \wedge \neg(c)$ ) because there exist prices  $\vec{p}$  and an interesting good  $G_k$  that is supplied in multiple, uncertainly ordered auctions, such that the buyer should buy good  $G_k$  but there are other prices  $\vec{q}$  that differ, in the future only, for which the buyer should buy the good in a future auction.  $\neg(b)$  implies the SAP (and even if  $(a) \wedge \neg(c)$ ) because there is at least one good, and one auction for that good, and two sets of prices that agree up until that auction but may disagree afterward, for which *even* if the buyer has acquired the optimal bundle of goods until this auction, as required by the eventual realization of future prices, the good in the current auction has an uncertain marginal value and therefore there is no single value the buyer can bid and ensure the right decision for all realizations of price. (c) implies the SAP (and even if  $(a) \wedge (b)$ ) because a buyer's bid may influence the bids of other buyers,

<sup>11</sup> The buying schedule is *temporally consistent* in the sense that  $IB_i(\vec{p}^{\leq k}) =^k IB_i(\vec{p}^{\leq l})$  for all  $l > k$ , all  $\vec{p} \in \mathcal{P}$ , where  $=^k$  requires that the first  $k$  elements are the same.

so that a buyer can do better than bidding her true marginal value for a good because this can trigger a high bid from a competitor in some future auction for some strategy of that competitor. ■

To understand this theorem, first consider a buyer who values one ton of sand for \$1,000 one ton of stone for \$2,000 and both for \$2,000 (i.e. substitutes valuations). Suppose the buyer faces one auction for sand followed by one for stone and that the possible prices allow for both goods to be utility maximizing. She faces the sequential auction problem because of condition  $\neg(b)$ : the sand has an uncertain marginal value because there are future prices for which it is optimal to purchase sand and some for which it is optimal to purchase stone. As a second example, consider a buyer who values one ton of sand for \$1,000 and one ton of sand and one ton of stone together for \$1,500. Suppose that the buyer faces an auction for sand and another for stone and that both bundles {sand} and {sand,stone} may be utility optimizing given possible prices. The good that first goes to auction has uncertain marginal value; this value is either \$1,000 or \$1,500 if the good is sand and \$0 or \$500 if the good is stone.

As another example, consider a buyer who values one ton of sand for \$1,000 and faces two auctions for sand, either of which may have the lowest price. She faces the sequential auction problem by condition  $\neg(a)$ ; here it occurs because even though the marginal value is \$1,000 and constant either auction may have the best price. For a final example, consider a buyer who values sand for \$1,000, stone for \$500 and sand and stone together for \$1,500 (i.e. a linear valuation) and suppose just two auctions, one for sand and then one for stone. Suppose one other buyer competes in both auctions. Both conditions (a) and (b) hold. But if condition (c) holds then the competitor may bid \$1,000,000 for stone if the buyer bids more than \$300 for sand, and \$10 otherwise. The buyer is better to bid below \$300 for sand, even if this involves losing that auction, because she will then receive the stone for a payoff of \$1,490.

### 3 An Options-Based Scheme

In what follows we focus exclusively on domains in which the underlying auctions are sealed-bid, Vickrey auctions for individual items. Vickrey auctions are selected not only because of convenience, but also because they nicely model eBay auctions. A Vickrey auction is a second price, sealed-bid auction.

The solution that we propose, in resolving the sequential auction problem in this context, consists of two primary components: the use of *real options* to allow buyers to secure the lowest possible prices and the use of *mandatory proxy agents* to prevent the abuse of these options through costless hoarding.

A real option is a right to acquire a real good at a certain price, called the *exercise price*; see Dixit and Pindyck [19]. For instance, Alice may obtain from Bob the right to buy sand from him at an exercise price of \$1,000. An option gives the *right* to purchase a good at an exercise price but does not imply an *obligation*. We will see that this flexibility makes options useful in addressing the sequential auction problem. Proxy agents acting on behalf of buyers can put together a collection of options, and then decide which options (perhaps none) to exercise.

While the buyer of an option has the *right* but not the obligation to purchase the good, the seller must honor the contract if the option is exercised. For this reason, options are typically sold at a premium called the *option price*. Several factors are often considered when a seller tries to determine how to price an option with a particular exercise price, including the relationship between the exercise price and the perceived value of the good available in the option, the volatility of value the good may experience over time, and the length of time over which a buyer can decide to exercise the option. Real options are often difficult to price as the metrics for determining a price are difficult to quantify. However, among *traded* options (i.e., options for traded securities such as stock), much progress has been made in determining the prices of options, with one of the most celebrated being the formula of Black and Scholes [5].

The problem with options with a non-zero option price in our setting is that they cannot support a simple, dominant bidding strategy because a buyer would need to compute the expected value of an option to justify the cost. But this expected value requires a probabilistic model of the future, which in turn requires the buyer to model the bidding strategies and values of other buyers. This is the very pattern of reasoning that we are trying to avoid in designing the options-based marketplace! For this reason, we adopt *costless options*, which always have an option price of zero. The exercise price is set competitively in the marketplace.

The traditional issue with costless options is that buyers are always weakly better off with a costless option than without one, whatever the exercise price. A buyer need exercise only those option(s) that result in a gain of surplus and bears no cost by not exercising an option. But having buyers that pursue options that they have no intention of exercising would cause market efficiency to unravel.

To prevent this kind of hoarding of options, we adopt mandatory proxy agents. These proxy agents provide buyers with an *obligation* to exercise those options that maximize their reported utility – as defined by reported valuation  $\hat{v}_i$  – given the exercise price. The proxy agents also act to restrict buyers to acquiring only options that they might credibly choose to exercise. Only proxy



agents place bids in the underlying auctions.

In Section 3.1 we give details about the proxied, options-based solution. Section 3.2 provides a number of detailed examples and Section 3.3 provides a complexity analysis. We delay until future sections a formal proof of strategyproofness and any analysis of the efficiency and revenue properties.

### 3.1 The Bidding Proxy and Price-Matching Rules

In our framework we first modify each individual Vickrey auction to sell a real option for the underlying good to the highest bidder with an *initial* exercise price equal to the second-highest bid price received.<sup>12</sup> Each option is costless, and is set to expire at the end of the winning proxy’s patience. Proxy agents bid in these auctions. In opting into the options-based protocol, a seller gives the winning proxy agent the right to reduce the exercise price on the option given evidence that a lower price would have been available had the proxy waited and bid instead in some future auction. This is what we mean by “price matching.”

Buyers must compete in the market by submitting a bid to their proxy agent. For buyer  $i \in B$ , this bid occurs in some reported arrival time,  $\hat{a}_i \geq a_i$ ,<sup>13</sup> and is a claim about her valuation  $\hat{v}_i$  (perhaps untruthful) for different bundles of goods and also about her departure time  $\hat{d}_i \geq \hat{a}_i$ . All transactions are intermediated by proxy agents. In what follows, we describe the three steps that are followed by a proxy agent: (a) acquiring options, (b) setting the exercise price on options via seller-sanctioned price matching, and (c) exercising options. This completely defines the options-based mechanism.

**Step One: Acquiring Options.** When an option pertaining to a good in which a buyer is interested is available in an auction and the proxy does not hold an option, then the proxy submits a bid equal to the buyer’s (reported) *maximum marginal value* for the item. A proxy does not bid for an item on which it already holds an option. The maximum marginal value for an item  $G_k$ , given reported valuation  $\hat{v}_i$ , is defined as:

$$bid_i(k) = \max_{L \subseteq G} [\hat{v}_i(L \cup \{G_k\}) - \hat{v}_i(L)] \quad (3.1)$$

<sup>12</sup> The system can also set a reserve price for each kind of good, provided that the same reserve is adopted for all auctions selling the same good. Without such an invariant reserve price, price matching would not be possible as a seller might be required to match a price below their personal reserve price.

<sup>13</sup> Recall that we assume that a buyer cannot submit a bid to her proxy until some period *after* her true arrival, which is why we have  $\hat{a}_i \geq a_i$ .

By bidding this value, a proxy will compete for any option that *could possibly* be of benefit to the buyer and choose not to bid only on those options that *could never* be of value to the buyer. Note that this is a static determination, made entirely in terms of the valuation of a buyer and without considering the prices for which the proxy already holds options. In bidding up to the maximal possible value of some item, the proxy is considering the case that all goods in bundle  $L$  that go together with this good will be available and for a price of \$0.<sup>14</sup>

**Step Two: Pricing Options.** Rather than acquire more than one option on the same kind of good, proxy agents are authorized by sellers to adjust the exercise price of an option that they win downwards. Such an adjustment is made whenever a proxy discovers that it could have achieved a better exercise price for an option on the same kind of good by waiting to bid in a later auction. A proxy is able to identify such a missed opportunity by storing locally, for each good on which it holds an option, *the identity of the active bidder (if any) that would have already won an option had the proxy itself not won an option*. For this, when a proxy first wins an option it stores in its local memory the identity (which may be a pseudonym) of the proxy agent that it “bumped” from winning, if such a proxy exists.

To see how price matching works, fix some good  $G_k$  on which the proxy holds an option. The proxy now monitors future auctions for the same kind of good and determines what the buyer population would be had the proxy delayed its own bid until this auction. To make this determination, the proxy requests from the auction the identities of the buyer proxies and their bids.<sup>15</sup> The proxy with the highest bid in this auction amongst those whose identity is *not* stored in the proxy’s local memory is exactly the buyer against whom the proxy would be competing had it delayed its entry until this auction. If the bid price of this other proxy is lower than the proxy’s current exercise price then it *price matches* down to this high bid amount, since this is exactly the price that the proxy could have achieved by delaying its bid until this auction.

After price matching, one of two adjustments are made by the proxy for

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<sup>14</sup> If the proxy has knowledge that some items will not be for sale, or can lower-bound the possible price on other items (e.g. because of a market-wide reservation price adopted by sellers – see Footnote 12), then the marginal value can be modified downwards to preclude such bundles or adjust downwards by lower bounds on the price of items. Care must be taken, though. We return to this issue in Section 6.

<sup>15</sup> In a marketplace such as eBay, this information could be provided (again in pseudonymous form) by the market infrastructure. Moreover, the only information that is minimally required is the highest bid price across all buyer proxies except one stated by the proxy, and the identity of highest proxy if it was not the winner of the auction.

book-keeping purposes. First, if the proxy whose price was matched against was the winner of the auction then the proxy's local memory as it relates to this good can be cleared. This is because the earlier bumped proxy must no longer be bidding (else it would have won this auction), and thus the proxy's earlier win no longer affects the set of active bidders for this good going forward. On the other hand, if the winning proxy was that which was earlier bumped, then the proxy that had the second highest bid in this auction – namely the proxy whose bid triggered price matching – would have won without the presence of the proxy in the market (because the earlier bumped proxy would have not competed in the current auction).

**Step Three: Exercising Options.** At the reported departure time,  $\hat{d}_i$ , the proxy for buyer  $i$  chooses which options (if any) to exercise. The option(s) that are exercised,  $L_i^*$ , are those that maximize the reported utility of the buyer given the final exercise price on each good:

$$L_i^* \in \operatorname{argmax}_{L \subseteq \mathcal{O}} [\hat{v}_i(\gamma(L)) - p(L)], \quad (3.2)$$

where  $\mathcal{O}$  is the set of all options held by the proxy,  $\gamma(L) \subseteq G$  are the goods that correspond to some subset  $L \subseteq \mathcal{O}$  of these options, and  $p(L) = \sum_{k \in L} p(k)$  is the total exercise price for this set of options where  $p(k)$  is the exercise price on the option corresponding to good  $G_k$ . All other options are returned. No options are exercised when there is no bundle of options with (weakly) positive utility.

**Remark: Re-posting seller options.** An auction for the good brought to market by a seller will first occur at the arrival period of the seller. If at some point later the buyer that wins this auction returns the option unexercised and the time period is before the seller's departure then it would be ideal to be able to initiate another auction for an option on the seller's good. However, the system prevents a seller from re-auctioning an option until the maximal patience,  $\Delta_{\max}$ , after the option was first allocated. Recall that  $\Delta_{\max}$  defines the maximal patience (departure-arrival) over all possible buyers in the market.<sup>16</sup> This maintains a truthful mechanism by preventing a buyer from acquiring an option with a view to holding it, returning it unexercised, and later re-entering the market when the option is again auctioned and achieving a lower price.<sup>17</sup>

<sup>16</sup> In practice one might choose to make a tradeoff here, allowing earlier reposting of an item in return for losing strategyproofness for all but the most patient buyers.

<sup>17</sup> Here is an example where Alice has a useful manipulation of this kind: Alice values an Apple for \$5 from Mon to Wed. Bob values the Apple for \$8 but only on Mon. Consider a seller with an Apple and high patience. If Alice is truthful then Bob wins on monday for \$5 and exercises the option. But if Alice claims to value an Apple together with a Banana at \$10 only from Monday to Tuesday then Alice wins the Apple option for \$8, but returns it at the end of Tuesday. On Wednesday,

Buyer	Reported Type	Monday	Tuesday
Molly	(Monday, Tuesday, \$8)	$6_{\text{Nancy}}$	$6_{\text{Nancy}} \rightarrow 4_{\text{Polly}}$
Nancy	(Monday, Tuesday, \$6)	-	$4_{\text{Polly}}$
Polly	(Monday, Tuesday, \$4)	-	-

Table 1

A 3 buyer example in which each buyer wants a single item and one auction occurs on each of Monday and Tuesday. “ $X_Y$ ” indicates an option with exercise price  $X$  and book-keeping that a proxy has prevented  $Y$  from currently possessing an option. “ $\rightarrow$ ” indicates the updating of exercise price, together with book-keeping for an option already held.

In the absence of *strong identities*, the market prevents buyers from affecting future supply in a useful way by waiting a sufficiently long amount of time before reauctioining a returned options.<sup>18</sup> However, in the presence of strong identities, the market can explicitly prevent a proxy’s buyer who has returned an option from bidding on that option for  $\Delta_{\text{max}}$  into the future. Consequently, a seller can reauctioan an option as soon as it is returned, increasing the likelihood of selling the item before her departure.

### 3.2 Examples of Proxy Behavior

We first provide an example to illustrate the price-matching logic that is followed by proxies:

**Example 3** *Consider three buyers, all of whom enter the market on Monday and depart the market after Tuesday. Molly values an item for \$8, Nancy for \$6 and Polly for \$4. On Monday, an auction occurs where all three proxies bid, with Molly’s proxy winning the highest bid of \$8 and receiving an option for \$6. Molly’s proxy adds Nancy to its local memory. On Tuesday, another auction occurs where only Nancy’s and Polly’s proxy bid, with Nancy’s proxy winning an option for \$4 and noting that it bumped Polly’s proxy. At this time, Molly’s proxy will price match its option down to \$4 (because Nancy is already in memory) and replace Nancy with Polly in its local memory for book-keeping purposes, as Polly would be holding an option had Molly delayed her bid past this round.*

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the seller re-posts and an Apple auction occurs with Alice returning and bidding \$5 for an Apple alone and winning the Apple for \$0.

<sup>18</sup> One common technique that is used at present to achieve strong, or almost strong, identities in electronic markets is to require a unique cell phone number of every registration.

To illustrate how the options-based scheme handles the exposure problem, consider the following example where Alice desires a bundle of two goods:

**Example 4** *Alice values one ton of sand and one ton of stone together for \$3,000 (but has no value for either by itself). Bob values one ton of sand for \$800. Charlie values one ton of stone for \$2,000. All buyers have a patience of 2 days. On day one, a stone auction is held, where Alice's proxy bids \$3,000 and Charlie's bids \$2,000. Alice's proxy wins an option to purchase stone for \$2,000. On day two, a sand auction is held, where Alice's proxy bids \$3,000 and Bob's bids \$800. Alice's proxy wins an option to purchase sand for \$800. At the end of the second day, Alice's proxy holds an option to buy stone for \$2,000 and sand for \$800 and exercises both options spending a total of \$2,800.*

As an illustration of how the options-based scheme handles substitutes values consider the following example:

**Example 5** *Alice values either one ton of coarse sand for \$1,000, or one ton of fine sand for \$800 (but only \$1,000 for both). Bob values coarse sand for \$800. Charlie values fine sand for \$900. On day one, a coarse sand auction is held where Alice's proxy bids \$1,000 and Bob's proxy bids \$800, resulting in Alice's proxy winning an option for the coarse sand with an exercise price of \$800. On day two, a fine sand auction is held where Alice's proxy bids \$800 and Charlie's proxy bids \$900, resulting in Charlie's proxy winning an option for the fine sand with an exercise price of \$800. At the end of day two, Alice's proxy exercises its coarse sand option and Charlie's proxy exercises its fine sand option.*

### 3.3 Complexity Analysis

In providing a complexity analysis for the problem facing proxy agents, we consider the particular case of a valuation function that is described in the *exclusive-or* (XOR) bidding language [39]. An XOR-valuation of size  $M$  defines a set of  $M$  atomic terms,  $\{(L^1, v_i(L^1)), \dots, (L^M, v_i(L^M))\}$ , and defines valuation  $v_i(S) = \max_{L^m \subseteq S, m \in \{1, \dots, M\}} [v_i(L^m)]$  for any bundle  $S$ , where  $L^m$  is one of the atomic terms. This is equivalent to saying that buyer  $i$  is interested in buying *at most one* of the atomic bundles.

We have the following two easy results:

**Theorem 1** *Given an XOR-valuation of size  $M$ , there is an  $O(K_i M^2)$  algorithm for computing the maximum marginal value on each interesting good for buyer  $i \in B$ , where  $K_i = |\cup_{m \in \{1, \dots, M\}} L^m|$  is the number of different*

items in which the buyer is interested.

**Proof.** For each item, recall Equation 3.1, which defines the maximum marginal value of an item. For each bundle  $L^m$  in the  $M$ -term valuation, and any item  $G_k$ ,  $v_i(L^m \cup \{G_k\})$  can be identified by considering each of the  $M$  terms in sequence. Therefore, the number of terms explored to determine the maximum marginal value for any item  $G_k$  is  $O(M^2)$ , and the total number of bundle comparisons to be performed to calculate the maximum marginal value on every item is  $O(K_i M^2)$ . ■

**Theorem 2** *The total memory required by a proxy to implement price matching is  $O(K_i)$ , where  $K_i = |\bigcup_{m \in \{1, \dots, M\}} L^m|$  is the number of different items in which the buyer is interested. The total work performed by a proxy in updating the state in each auction is  $O(1)$ .*

**Proof.** The proxy stores one maximum marginal value for each item of interest, of which there are  $O(K_i)$ ; at most one buyer's identity for each item, of which there are  $O(K_i)$ ; and one exercise price for each such item, of which there are  $O(K_i)$ . For each auction, the proxy either submits a precomputed bid or price matches, both of which take constant work. ■

## 4 Theoretical Analysis of the Options-Based Scheme

In this section, we establish that the options-based scheme supports truthful bidding as a dominant bidding strategy for buyers and whatever the strategy of sellers. We also develop a worst-case, competitive analysis for allocative efficiency in the special case in which all buyers and sellers trade units of the same good. This competitive analysis generalizes earlier analysis due to Hajiaghayi et al. [25] to include a bound on the maximum ratio of minimum to maximum values in the buyer population. This is a useful modification because values will typically fall into natural bounds in practical settings.

### 4.1 Strategic Analysis: Establishing Truthfulness

The proxies transform the market into a direct revelation mechanism, where each buyer  $i$  makes a claim about her type to her proxy agent in some period  $\hat{a}_i \geq a_i$ . We will show that it is a dominant strategy for a buyer to reveal her *true* valuation and *true* departure time to her proxy agent immediately upon arrival to the system, that is to bid her true type  $\theta_i = (a_i, d_i, v_i)$ . We assume for technical reasons that the market is “opaque” so that no buyer receives any information about prices or bids in the marketplace before submitting a

bid to her proxy agent.<sup>19</sup> Note that this opaqueness is *not* required between proxy agents and the market. On the contrary, it is vital that proxy agents receive information about bids from other proxies in order to perform price matching.

In establishing strategyproofness, we provide a slight generalization of an existing characterization of strategyproof online auctions [25], to allow for combinatorial online auctions. For this, define a *value-independent price function*,  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L) \geq 0$ , on all  $L \subseteq G$ , which can depend on the bids of other agents  $b_{-i}$  and the reported arrival  $\hat{a}_i$  and departure  $\hat{d}_i$  of buyer  $i$  but is value-independent in that it does not depend on the reported valuation  $\hat{v}_i$  of the buyer. The price function also depends on the realization of supply, but this dependence is suppressed to keep the notation simple.

**Definition 2 (monotonic prices)** *A value-independent price function is monotonic in arrival and departure if  $ps_{a'_i, d'_i, b_{-i}}(L) \geq ps_{a_i, d_i, b_{-i}}(L)$ , for all buyers  $i$ , all  $a'_i \geq a_i$ , all  $d'_i \leq d_i$ , all bids  $b_{-i}$  by other buyers, all realizations of supply, all  $a_i, d_i$  and all  $L \subseteq G$ .*

Monotonic prices increase with a tighter arrival-departure interval, or a larger set of goods, and whatever the bids from other buyers and whatever the realization of supply.

**Lemma 1** *An online combinatorial auction is strategyproof in a setting with limited misreports, so that buyers can only report later arrivals  $\hat{a}_i \geq a_i$ , if there exists a monotonic, value-independent price function,  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L)$ , such that for every buyer  $i$ , all reports  $\hat{\theta}_i = (\hat{a}_i, \hat{d}_i, \hat{v}_i)$ , all bids  $b_{-i}$  from other buyers, and all realizations of supply, the buyer is allocated a bundle  $L_i^* \in \operatorname{argmax}_{L \subseteq G} [v_i(L) - ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L)]$  in period  $\hat{d}_i$  and makes payment  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L_i^*)$ .*

**Proof.** Fix some  $a'_i$  and  $d'_i$ . A buyer should report true valuation function,  $\hat{v}_i = v_i$ , because the prices she faces are independent of her report and by being truthful bundle  $L_i^*$  maximizes her true utility. This in place, fix  $\hat{v}_i = v_i$ . Now, it is never useful to bid  $\hat{d}_i > d_i$  because the buyer will not receive her allocation until her true departure (and have zero value.) By limited misreports, the buyer cannot report  $\hat{a}_i < a_i$ . Reporting  $\hat{a}_i > a_i$  or  $\hat{d}_i < d_i$  (weakly) increases the price on every bundle  $L$  by monotonicity. ■

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<sup>19</sup> The impact of allowing buyers to see information about prices and bids would be to change the equilibrium from a dominant strategy equilibrium to an *ex post Nash* equilibrium, in which truthful bidding would remain an equilibrium for all possible private types of other buyers and all possible futures as long as other buyers are also rational and bid truthfully. The analysis is not changed in any substantive way.

We construct a monotonic, valuation-independent price function for each buyer in terms of the final exercise prices of options held by the buyer's proxy agent and establish that the proxy agent maximizes the buyer's reported valuation function given these prices. The following lemma is useful in establishing the correctness of the book-keeping algorithm:

**Lemma 2** *At any given time, for any buyer  $i$ , there is at most one other buyer in the system whose proxy does not hold an option for a given item because of buyer  $i$ 's presence, and the identity of that buyer will be stored in buyer  $i$ 's proxy's local memory at that time if such a buyer exists.*

**Proof.** Fix some item and the proxy for buyer  $i$ . The proof is by induction on the sequence of auctions for the item while buyer  $i$ 's proxy is present in the market. The correctness of the information in the proxy's local memory is easy to establish in the base case before the proxy has participated in any auction. Now consider the first auction for this item in which the proxy wins and suppose it prevents another proxy from winning an option on the good. (This is the interesting case.) Consider now two cases: (a) the bumped proxy will leave the system having never won an option on the item, or (b) the bumped proxy will win an auction on this item in the future. In case (a), while this bumped proxy is still present then proxy  $i$ 's presence prevented exactly that one proxy but no other proxies from winning an option. This is because the presence of the bumped proxy in the market does not preclude any other proxy from winning (because the bumped proxy is losing anyway.) The identity of this bumped proxy remains in proxy  $i$ 's local memory because no price matching will have occurred on this item because each winning proxy in each subsequent auction must have submitted a bid higher than the bumped proxy's bid (else the bumped proxy would have won) and therefore higher than proxy  $i$ 's exercise price, which is initialized to the bid price of the bumped proxy. Eventually the bumped proxy leaves and proxy  $i$  no longer has any effect on the bid dynamics for this item. At some point some other proxy may win on this item and the proxy's local memory is cleared. In case (b), while the bumped proxy is not yet winning then this is as in case (a). In period  $t$  in which the bumped proxy wins then proxy  $j$ , with the highest other bid in that auction (if any), would have won without  $i$ 's presence. Proxy  $i$  necessarily price matches in this case – because the exercise price it could have achieved is the bid price of proxy  $j$  and less than that of the winning (bumped) proxy and thus its current exercise price – and then updates its local memory to contain the identity of proxy  $j$ . Proxy  $j$  is the new proxy that does not hold an option because of buyer  $i$ 's presence in the market. Case (a) or (b) now holds again for this new bumped proxy, proxy  $j$ , and the proof continues until proxy  $i$  finally departs the market. ■

In proving the strategyproofness of the auction, we will consider the special case of a function  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L)$  that is *linear* in the items  $G_k \in L$ , so that



$ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L) = \sum_{G_k \in L} ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k)$ , where price function  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k)$  on items is itself monotonic in arrival and departure. The following easy lemma is stated without proof:

**Lemma 3** *The bundle of goods that maximizes a buyer's reported valuation given a linear, agent-independent and monotonic price function, defined in terms of the sum of prices on individual goods, will never contain an item that is priced above her maximum marginal value.*

**Theorem 3** *Truthful bidding is a dominant-strategy equilibrium for buyers in the options-based, proxied market in a setting with limited misreports, so that buyers can only report later arrivals  $\hat{a}_i \geq a_i$ .*

**Proof.** Fix buyer  $i$ . First, define an agent-independent price,  $p_{b_{-i}}^t(k)$ , on item  $G_k$  in period  $t$  as the highest bid by the proxies  $\neq i$  not holding an option on item  $G_k$  at time  $t$  ( $\infty$  if there is no supply at  $t$ ), and not including any proxy that would have already won an option had  $i$  never entered the system (i.e., whose identity is stored in the  $i$ 's proxy's local memory for item  $G_k$  by Lemma 2). Conditioned on  $t \geq \hat{a}_i$ , this price is well defined and independent of any report  $\hat{\theta}_i$  buyer  $i$  makes to her proxy because the set explicitly excludes the at most one proxy (see Lemma 2) that  $i$  prevents from holding an option by its presence and is exactly those bids that would be made without  $i$  present. (For this, we leverage that the bid values are equal to maximal marginal values and independent of earlier bids by buyer  $i$ 's proxy.) Furthermore,  $i$  cannot influence the supply of item  $G_k$  because any options returned by other buyers due to a price set by  $i$ 's proxy's bid will be re-auctioned (if at all) after  $i$  has departed the system. Now define  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k) = \min_{\hat{a}_i \leq \tau \leq \hat{d}_i} p_{b_{-i}}^\tau(k)$  (possibly  $\infty$ ), which is well defined upon  $\hat{d}_i$  and is monotonic in arrival and departure because it is defined as the minimal value over its domain. Conditioned on holding an option on  $G_k$  upon departure, this is exactly the exercise price obtained by buyer  $i$ 's proxy. Now define  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L) = \sum_{k: G_k \in L} ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k)$ , which is monotonic in  $\hat{a}_i$  and  $\hat{d}_i$  because  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k)$  is monotonic and remains value-independent. (Note that this price is  $\infty$  when there was never any supply of item  $G_k$ .) Given the options held by a proxy at  $\hat{d}_i$ , which may be a subset of those items  $G_k$  with prices  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k) < \infty$ , the proxy exercises options to maximize utility based on reported valuation,  $\hat{v}_i$ . We show that the proxy would not want to select any options on items that are not available because the prices on missing options would be too high. For this, consider such a bundle  $L'$ , that is interesting based on  $\hat{v}_i$  but has one or more missing options. One possibility is that there is an item  $G_k \in L'$  for which an option was never for sale, in which case  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k) = \infty$  and  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L') = \infty$  and  $L'$  is not utility maximizing. On the other hand, in the case that every item in  $L'$  was available for sale in interval  $[a_i, \dots, d_i]$ , we know that  $p_{b_{-i}}^t(k)$  was at least the

maximal marginal value in every such period (else the buyer’s proxy would have won) and the bundle cannot maximize utility by Lemma 3. ■

A proxy acquires an option on every item that could possibly be in a buyer’s utility-maximizing bundle at the final prices. For any option that a proxy agent does not explicitly hold upon departure, either the item was never for sale, *or* the competition was such that the price was always so high that the buyer would not want to exercise the option even if the clearing price on the other options was zero.

**Remark 1.** The options-based scheme also satisfies *voluntary participation* (or “individual rationality”) for both buyers and sellers, meaning that every participant has non-negative utility in equilibrium. For buyers, this follows because the proxy exercises a utility maximizing set of options and will exercise no options if all bundles have negative utility. For sellers without any intrinsic value for the good, voluntary participation follows because the prices on options remain non-negative.

**Remark 2.** The options-based scheme is robust against buyers that can participate under multiple identities or through re-entry with the same identity. This is because the price function  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(L)$  is *linear* on every bundle of goods  $L \subseteq G$ , being defined as the sum of the prices,  $ps_{\hat{a}_i, \hat{d}_i, b_{-i}}(k)$  introduced in the proof. So long as the prices that a buyer faces remain unchanged when she participates multiple times, she cannot gain by winning multiple bundles because of linearity of prices and because all alternate bids must necessarily have (weakly) tighter arrival-departure intervals and therefore higher prices. In fact, the prices may increase – but cannot decrease – when a buyer participates multiple times because prices in the underlying auctions are weakly increasing in more participants (a property of the Vickrey auction), and note that the supply available to a buyer is unaffected by its strategy because of the delay that is required of a seller before reposting an item for auction.

## 4.2 Efficiency Analysis

We provide a worst-case, competitive analysis in the special case in which all participants are buying and selling one unit of the same kind of good. An online mechanism is said to be *k-competitive with respect to efficiency* if it is guaranteed to achieve an allocation with value at least  $1/k$  of that achieved in an optimal *omniscient* allocation. The omniscient allocation maximizes total value given perfect hindsight about the arrival and departure and values of market participants. For example, a mechanism that is 2-competitive for efficiency will achieve a total value that is at least half of

the total value of the optimal, offline allocation, for all possible inputs.

In this special case in which all buyers and sellers trading one unit of the same good, our problem is the same as that considered by Hajiaghayi et al. [25].<sup>20</sup> These authors establish that their mechanism (and thus also our options-based scheme) is 2-competitive for efficiency and prove a tight lower-bound to show that no truthful, online mechanism can provide better than 2-competitiveness. In this sense, we are comforted to see that our scheme has the best possible, worst-case efficiency in this setting.

We provide a slight generalization of this analysis in which we parameterize the competitive analysis with a lower bound,  $0 \leq \alpha \leq 1$ , on the ratio of the minimum to maximum value for the item in the buyer population; i.e.  $w_i/w_j \geq \alpha$  for all buyers  $i, j \in B$ , with values  $w_i, w_j$  on the item for buyers  $i$  and  $j$  respectively. For  $\alpha = 0$  there is no bound, for  $\alpha = 1$  then all buyers have the same value. This additional constraint is relevant in modeling an eBay like domain in which the inputs are not adversarial but instead drawn from some distribution.

**Theorem 4** *When every buyer and seller is interested in trading one unit of the same item, there is a lower-bound  $0 \leq \alpha \leq 1$  on the ratio of minimum to maximum values in the buyer population, and when all items are sold in the options-based scheme (at least for  $\alpha > 0$ ), then the options-based scheme is  $\frac{2}{1+\alpha}$ -competitive for efficiency.*

**Proof.** Let  $OFF$  and  $ON$  denote the winners in the offline and online solutions respectively. We seek a lower bound  $V_{ON}/V_{OFF} = \sum_{i \in ON} v_i / \sum_{i \in OFF} v_i \geq 1/k$  for all possible inputs. For any input, we can upper bound  $V_{OFF}$  in terms of  $V_{ON}$  through a charging argument. Following Hajiaghayi et al. [25], consider some buyer  $i \in OFF$ . We “charge” her value to a buyer in  $ON$ . If  $i \in ON$  then we charge the value to herself. Otherwise, let  $auc$  be the auction that  $i$  wins offline. Since  $i$  never wins in the options-based market, she was present in the market when  $auc$  closed, and so the options-based scheme must have picked a winner  $j \in ON$  whose value is (weakly) greater than the value of  $i$ . We charge the value of  $i$  to  $j$ . It is not hard to see that this charging scheme charges each agent  $j$  in the options-based market at most twice, each time for a value less than the value of  $j$ . Let  $ONCE \subseteq ON$  and  $TWICE \subseteq ON$  denote the online winners that are charged once and twice respectively. We have that  $V_{OFF} \leq \sum_{j \in ONCE} v_j +$

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<sup>20</sup> These authors propose a mechanism that combines a greedy matching algorithm, in which the next item is allocated to the agent with the highest value that is currently unmatched, and collects as payment from a winner the “critical value,” which is the smallest value that the buyer could have reported and still been successfully matched. It is easy to show that the options-based scheme presented here is *equivalent* to this greedy, critical-value based mechanism in this special case.

$2 \sum_{j \in TWICE} v_j$ . For  $\alpha = 0$  this gives  $\frac{V_{ON}}{V_{OFF}} \geq \frac{(\sum_{j \in ONCE} v_j + \sum_{j \in TWICE} v_j)}{(\sum_{j \in ONCE} v_j + 2 \sum_{j \in TWICE} v_j)} \geq 1/2$ , with the worst-case occurring for  $ONCE = \emptyset$ . Consider now  $\alpha > 0$ , and let  $B' \subseteq OFF$  denote the subset of  $OFF$  that are matched to the  $TWICE$  set. Let  $K = |B'|$  and note that  $K$  must be even so that  $K/2$  is an integer. Now, for  $\alpha > 0$  we know that  $n = |OFF| = |ON|$ , because all items are sold in the online solution by assumption. Then, we have  $|ONCE| = n - K$ ,  $|TWICE| = K/2$ , and an additional  $n - (n - K) - K/2 = K/2$  winners in  $ON$ . Let  $\bar{V}$  denote the maximal value across all winners in set  $OFF$ . We have  $V_{ON} \geq \sum_{j \in ONCE} v_j + \sum_{j \in TWICE} v_j + (K/2)\alpha\bar{V}$ , and therefore

$$\frac{V_{ON}}{V_{OFF}} \geq \frac{\sum_{j \in ONCE} v_j + \sum_{j \in TWICE} v_j + (K/2)\alpha\bar{V}}{\sum_{j \in ONCE} v_j + 2 \sum_{j \in TWICE} v_j} \quad (4.1)$$

$$\geq \frac{(n - K)\alpha\bar{V} + (K/2)\bar{V} + (K/2)\alpha\bar{V}}{(n - K)\alpha\bar{V} + K\bar{V}} \quad (4.2)$$

$$= \frac{\alpha(n - K) + (K/2)(1 + \alpha)}{\alpha(n - K) + K} \quad (4.3)$$

$$\geq \frac{K/2(1 + \alpha)}{K} = \frac{1 + \alpha}{2}, \quad (4.4)$$

where the second inequality follows by substituting the smallest possible values of agents in  $ONCE$  (counted equally in numerator and the denominator) and the largest possible values of agents in  $TWICE$  (counted twice in the denominator). The final inequality follows by analysis of the rate of change of the numerator and the denominator with respect to  $K$  for any  $\alpha \in (0, 1]$ , which is always more positive for the denominator than the numerator and therefore a valid lower bound is achieved by setting  $K = n$ . ■

When  $\alpha = 0$ , that is with no prior assumption about the possible range of buyer values, we have exactly the competitive ratio of 2. The competitiveness goes to 1 as  $\alpha \rightarrow 1$ , as the values become more and more homogeneous. In our analysis of the eBay LCD market, we find the values of all buyers are bounded between \$200 and \$300, in which case  $\alpha = 0.67$  and the competitive ratio for efficiency is around 1.2, which implies that the total value of the allocation made by the options-based scheme is guaranteed to be within 17% of the value of the best-possible offline solution.

## 5 Empirical Analysis

In this section, we present the results of an experimental study of the *average-case* performance of the options-based scheme for both efficiency and revenue. This study is in two parts. We first report results from an analysis of data collected from eBay on all auctions for a 19" Dell LCD monitor sold

during the summer of 2005. From this data we derive a population of buyers and sellers, including estimates of the (true) arrival and departure times, and true values of buyers. We adapt a non-parametric approach to estimate the values of buyers (Haile and Tamer [24], extended to dynamic auctions in Juda [31]), and couple this with bootstrapping to provide robustness. We estimate that the options-based scheme would provide a 4% improvement in efficiency and a 9% improvement in revenue over the status quo.

We also report the results from additional simulations designed to understand the performance of the options-based scheme in environments in which buyers have *substitutes* valuations or *complements* valuations. These simulations are inspired by the eBay data but are not directly performed in terms of this data because we do not attempt to identify the preferences of buyers with more complex valuations.<sup>21</sup> Buyer populations with substitutes valuations can hamper efficiency, although efficiency remains high when the valuations of a given buyer for the different items are either negatively correlated or uncorrelated. Buyer populations with complements valuations tend to achieve consistent efficiencies for different within-buyer correlation on the value of items. Finally, we study the role of liquidity by varying the buyer-to-seller ratio and find that for buyer-to-seller ratios that are typical in the eBay marketplace the efficiency remains high even when buyers have substitutes valuations over many different items.

### 5.1 An eBay Market for LCD Screens

We collected data from eBay on all auctions for a 19" Dell LCD monitor (Model E193FP) sold from 27 May, 2005 through 1 October, 2005, of which there were 1,956 instances.<sup>22</sup> Assuming that each pseudonym represents a

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<sup>21</sup> When buyers have substitute preferences, it would be difficult to determine the entire set of substitutes in which a buyer may be interested, as a buyer on eBay may have never bid on all substitutes. When buyers have complementary preferences, it would be difficult to determine based solely on their bidding behavior the extent to which they already possess the complementary goods. For example, if we were to observe a buyer bidding on a left shoe, there is no way to know definitively if the buyer already possesses a right shoe, or only intends to start bidding on a right shoe once a left shoe has been acquired.

<sup>22</sup> Specifically, search queries found all auctions where the auction title contained all of the following terms: 'Dell,' 'LCD' and 'E193FP,' while excluding all auctions that contained any of the following terms: 'Dimension,' 'GHZ,' 'desktop,' 'p4' and 'GB.' The exclusion terms exist so that the only auctions analyzed would be those selling exclusively the LCD of interest. For example, the few bundled auctions selling both a Dell Dimension desktop and the E193FP LCD are excluded. Further information on the fields for each auction and how those fields were processed is provided in

unique buyer, we observe 10,151 distinct bidders participating in these auctions. Given the data, our aim is to simulate a sequence of auctions for options that match the timing of auctions on eBay and with the true, underlying value of the buyers as identified from the eBay data. For eBay revenue we take the revenue as defined by the actual closing prices in the data. For eBay efficiency we compute the efficiency implied by the allocation on eBay and the estimate of the true values that we make for each winner.

For each auction that closes on eBay with a sale, we simulate a Vickrey auction for an option on the item. Auctions on eBay in which the item goes unsold are not considered within our simulation.<sup>23</sup> The sequence in our simulation is the same as the sequence on eBay with an auction scheduled to occur when it first *opens* on eBay. An auction that opened at 1:00:00pm on day 1 would be simulated before an auction that opened at 1:00:01pm on day 1. We schedule auctions at the time an auction opens rather than closes on eBay to allow for the possibility of re-posting an item that goes unsold. (Although this is only relevant with more general valuations because all options are exercised in the current context.)

We estimate the arrival, departure and value of each buyer on eBay from their observed behavior.<sup>24</sup> Arrival is estimated as the *first time that a buyer interacts with the eBay proxy*, while departure is estimated as the *latest closing time among eBay auctions in which a buyer participates*. Both are clearly conservative, but adopted in the interest of simplicity. We note that less conservative estimates of these timing constraints would improve the performance of the options-based auction in simulation because there would be greater competition. We are careful about the end effects of the first few days of data and the last few days of data.<sup>25</sup>

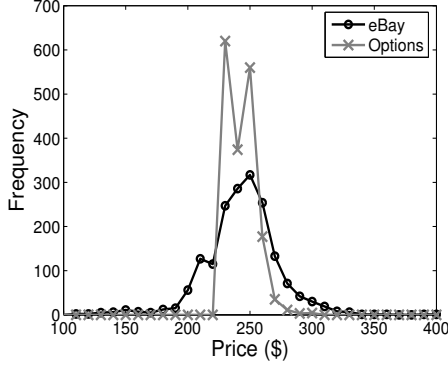
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Juda [31].

<sup>23</sup> Unsuccessful auctions are likely either completely unseen by the buyer population or reserve auctions where the reserve price was not met. In either scenario, we consider these auctions too unique to include in the simulation. For example, if unsuccessful auctions were modeled in the options-based scheme, then more items would be sold in the options scheme than on eBay, making it significantly more difficult to compare the total revenue generated between the two markets.

<sup>24</sup> The relatively few buyers observed to have won multiple items on eBay in practice are simulated as multiple buyers with identical arrival, departure and value. At most one of these identical buyers will participate in any given auction.

<sup>25</sup> When running the simulations, the final ten days worth of observed auctions are not simulated. Ten days also is past the 90-percentile of the distribution of buyer patience. Buyers bidding toward the end of the window are likely to bid higher in the future on eBay than their current bids (this is suggested by the regression analysis in Juda [31]), which results in inaccurately low estimates for buyer value among these buyers. Similarly, we must allow for a start effect in which the initial revenue in simulation will be artificially high because no buyers in the initial period will be marked as already traded when in fact they would have if the simulation



(a) The probability density function for revenue across sellers of the Dell E193FP LCD screens using worst-case estimates of buyer values. While the average closing price on eBay, and in the options scheme, are comparable, the variance is significantly lower in the options scheme.

	Options	eBay
$\overline{Price}$	\$239.66	\$240.24
$stddev(Price)$	\$12	\$32
$\overline{Value}$	\$263	\$244
$\overline{BuyerSurplus}$	\$23	\$4

(b) The average price paid per good, average buyer value among winners, and average winning buyer surplus on eBay for Dell E193FP LCD screens as well as the simulated options-based market using worst-case estimates of buyer values.

Fig. 1. Comparisons between empirically observed results on eBay and simulation results of the options scheme using a worst-case estimate of buyers’ true valuations.

In our first experiments, we adopt a conservative estimate of the true value of a buyer on eBay, estimating this simply as the *highest bid this buyer was observed to have placed in any LCD auction*. Figure 1(a) shows the distribution of closing prices both on eBay and in the simulated options scheme under this “worst-case” assumption. The closing price in the simulation is defined as the final exercise price (i.e., after price matching.) The revenue is similar between the two schemes while the estimated effect on efficiency suggests an advantage for the options-based scheme. See Figure 1(b). While the winning buyers on eBay have an estimated mean value of \$244, the options scheme’s winning buyers have a mean value of \$263 (a 7% increase). Consumer surplus, which measures the average buyer utility over all winning buyers, increases in the options-based scheme from \$4 to \$23 (an improvement of roughly 500%). Notice also that the standard deviation of prices is significantly reduced in the options-based scheme.

Figure 2 shows a very distinct difference between eBay and the options scheme. We plot the closing price of an auction against the patience of the auction’s winner. In the options scheme, not only do buyers with a larger patience generally pay lower prices than buyers with smaller patience but the variance of price paid decreases with patience.

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had started earlier. For this, the options simulation starts from the initial period but revenue is only accounted after the first ten days.

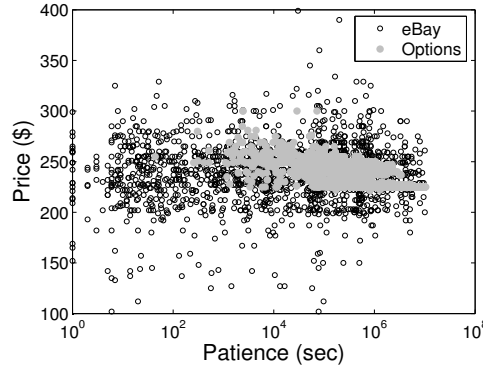


Fig. 2. A scatter plot of the price paid by winners against the patience of the buyer (in secs) for Dell E193FP LCD screens using worst-case estimates of buyer values.

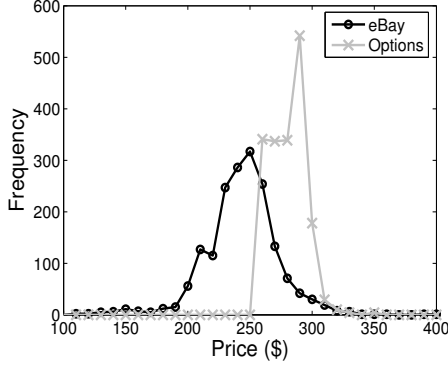
In our second set of experiments we adopt a less conservative estimate of the true value of a buyer by using the non-parametric methods of Haile and Tamer [24], generalized to apply to dynamic auctions.<sup>26</sup> Figure 3(a) shows the distribution of actual closing prices in eBay and in the options scheme as simulated with this new, less conservative estimate. The mean price in the options scheme is now significantly higher than the mean price on eBay (\$240 on eBay, \$276 in the options scheme). The standard deviation on closing prices in the options scheme is also significantly less, being \$32 on eBay vs. \$14 in the options scheme. The estimated efficiency of the options-based scheme remains higher than that on eBay. While the winning buyers on eBay are estimated to have a mean (true) value of \$281, the winners in the options scheme are estimated to have a mean value that is 7.5% higher (at \$302).<sup>27</sup> It bears emphasis that in reporting these results, the *same* value estimates that are adopted in the simulation of the options scheme are also adopted in estimating the total value of the allocation implemented on eBay.

**Bootstrapping** While the simulation of the options-based market suggests better efficiency and revenue results performance than on eBay, a reasonable concern may be that the performance of the options-based market is influenced by specific details of the particular buyer population considered. To alleviate these concerns, we also perform a set of bootstrapped simulations. Rather than using the 10,151 unique buyers observed to have

<sup>26</sup> See Juda [31] for more information. In particular, we are able to estimate that the true values of buyers on eBay are 15% greater than their observed maximum bids. This estimate is based on identifying a multiplicative factor that separates the distribution of observed bid values from a conservative estimate on the distribution of underlying values that can be derived through analysis of order statistics and simple, reasonable assumptions.

<sup>27</sup> The consumer surplus for buyers in the options scheme is estimated to be slightly below that on eBay, but this could be easily addressed by the use of fees or other methods to redistribute surplus.





(a) Revenue distribution among auctions on eBay and in the simulated options-scheme for a less conservative estimate of buyer values.

	Options	eBay
$\overline{Price}$	\$275.80	\$240.24
$stddev(Price)$	\$14	\$32
$\overline{Value}$	\$302	\$281
$\overline{BuyerSurplus}$	\$26	\$40

(b) The average price paid per good, average buyer value among winners, and average winning buyer surplus on eBay and under the simulated options market, for Dell E193FP LCD screens and a less conservative estimate of buyer values.

Fig. 3. Comparisons between empirically observed results on eBay and simulation results of the options scheme using a less conservative estimate of buyers' true valuations.

	Options	$\frac{Bootstrap}{Options}$	Bootstrap	$\frac{Bootstrap}{eBay}$	eBay
$\overline{Price}$	\$275.80	0.95	\$261.89 (\$1.37)	1.09	\$240.24
$\overline{Value}$	\$302	0.97	\$292 (\$0.86)	1.04	\$281
$\overline{BuyerSurplus}$	\$26	1.14	\$30 (\$0.95)	0.74	\$40

Table 2

Average results of 50 bootstrapped simulations of the options-based scheme (standard deviations in parentheses), compared to the options-based scheme without bootstrapping and eBay.

participated on eBay in the simulation we instead simulate the options-based market using 10,151 buyers where each buyer is drawn uniformly with replacement among all buyers observed to have bid on eBay. This creates a buyer population that is similar to that observed on eBay, while providing insight into how sensitive the results are to the exact combination of buyers observed. Table 2 provides the average result of 50 bootstrapped simulations, together with the performance of the options-based scheme without bootstrapping and the results for the actual allocation in the eBay market. The results support an estimate of efficiency that is 4% greater than on eBay and revenue that is 9% greater than on eBay.

## 5.2 Simulation: Substitutes Preferences

While the options-based market appears effective when the bidding population has simple preferences and wants only a single good, we now examine the efficacy of the system when buyers have substitutes preferences. To see why substitutes preferences can be problematic consider the following:

**Example 6** *Alice values one of either an apple or banana by Tuesday for \$10. Bob values one of either an apple or banana by Tuesday for \$8. On Sunday, an apple auction is held where Alice’s proxy wins an option for the apple for \$8. On Monday, a banana auction is held where Alice’s proxy wins an option for the banana for \$8. At the end of Tuesday, Alice’s proxy exercises one of her two options, returning the other option, while Bob leaves the market having acquired nothing.*

Clearly, it would have been more efficient for Bob to have won an option for only one of the pieces of the fruit. However, the scheme has Alice’s proxy holding both options. We refer to this as the *holdup problem*.<sup>28</sup>

We first consider a market in which buyers have substitutes preferences over two items. Inspired by the observed population of eBay, we consider a market with a 120-day time period where each buyer’s value is distributed normally over a Gaussian distribution for Monitor A with mean \$265 and standard deviation \$45 and for Monitor B with mean \$240 and standard deviation \$20. The value for the bundle of A and B is the maximum of the two values. 5,000 buyers arrive uniformly over the 120-day time period, and with patience distributed according to a Normal distribution with a mean of 3.9 days and a standard deviation 11.4 days (as was observed to be the mean and standard deviation of patience among the eBay buyers), and truncated to be non-negative and rounded to the nearest day. We model 2,000 sellers that enter the market uniformly over the 120-day time period, with the patience of each seller distributed by a Normal distribution with a mean of 7 days and a standard deviation 1 day. Each seller offers one of Monitor A or Monitor B with equal probability. In simulation, an auction is scheduled for a seller for an option on her good immediately upon arrival and the good is re-posted when a seller’s option is returned unexercised before her departure.<sup>29</sup>

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<sup>28</sup> To provide a counterpoint, note that if buyers have linear-additive values on individual items then they will always exercise every option they win and no options will be returned and go unsold. This is because the maximum marginal value of each item is exactly the value a buyer will ultimately realize in exercising that option, whatever the details of the other goods that it wins.

<sup>29</sup> For this to be possible in practice without introducing new opportunities for buyer manipulation we would need to prevent a buyer’s proxy from rebidding on the same

Buyers’ Values	Items Sold (%)	Total Value	Buyer Surplus (%)	Sellers Returned (%)	Efficiency (%)
- Correlated	100.0	569,665	7.6	0.02	96.5
Uncorrelated	87.1	500,571	8.5	18.3	85.2
+ Correlated	61.4	362,704	9.2	41.7	63.3

Table 3

Market performance (averaged over 30 instances) with 5,000 buyers and 2,000 sellers in a 120 day marketplace and buyers with substitutes preferences on 2 items.

In the experiments, we consider both a positive and a negative correlation between the value that a buyer has for Monitor A and Monitor B. Positive correlations might exist if some buyers possess generally higher valuations across all items than other buyers. Negative correlations suggest that buyers have strong “tastes” for each item and it likely that either one or the other item will appeal to a particular buyer but not both.<sup>30</sup>

Table 3 shows summary statistics for the performance of the market in this setting. Rather than provide a comparison to eBay, we compare to the efficiency that would be possible in an omniscient solution (found here as the solution to a mixed-integer program). The “sellers returned” statistic indicates the average number of sellers whom have an option returned unsold. One can understand by comparing the fraction of items sold and the fraction of “sellers returned” the number of items that are sold successfully on second (and later) attempts; e.g., for uncorrelated values 18.3% of items are initially unsold but only  $100 - 87.1 = 12.9\%$  of items are unsold eventually.

When values across the two items are negatively correlated the market in effect breaks itself up into two disjoint markets, one for the first and one for the second item, because buyers do not typically possess a sufficiently high value on both of the items to be competitive on both. Because of this all items are typically sold and the efficiency is high and there is only a slight holdup problem. On the other hand, when values across the two items are either uncorrelated or positively correlated, buyers are more likely to hold options on both items, thus causing holdup problems and blocking lower-valued buyers who may never hold an option.

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item within period  $\Delta_{\max}$ , as discussed in Section 3.1.

<sup>30</sup> To model positive correlation, if a buyer’s valuation for Monitor A is  $x$  standard deviations above the mean, her valuation for Monitor B is set to  $x$  standard deviation above the mean (c.f. for  $x$  standard deviations below the mean). Alternatively, to model negative correlation, if a buyer’s valuation for Monitor A is  $x$  standard deviations above the mean, her valuation for Monitor B is set to  $x$  standard deviations below the mean (c.f. for  $x$  standard deviations below the mean).

Buyers' Values	Items Sold (%)	Total Value	Buyer Surplus (%)	Sellers Returned (%)	Efficiency (%)
Uncorrelated	75.0	632,504	7.5	33.8	74.5
+ Correlated	44.4	388,191	8.6	58.1	47.2

Table 4

Market performance (averaged over 30 instances) with 5,000 buyers and 3,000 sellers in a 120 day marketplace. Buyers possess substitutes preferences over 3 items.

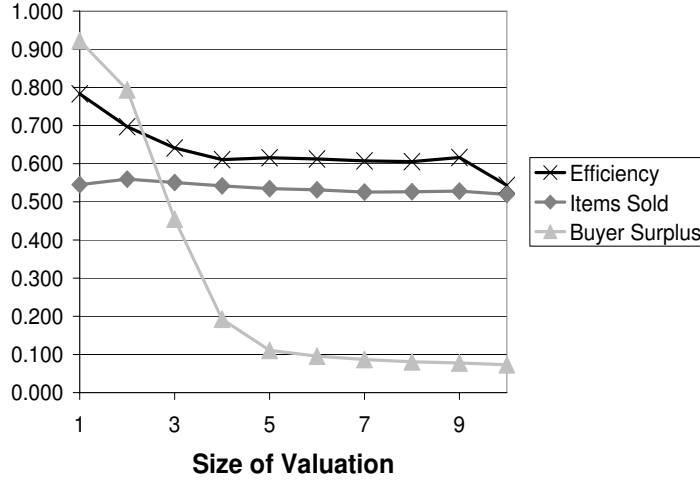


Fig. 4. Market performance (averaged over 30 runs) with 1,000 buyers and 1,000 sellers in a 120 day marketplace with 10 different kinds of goods being offered. Buyers have substitutes preferences over a varying number of items.

Similar results are found when we consider substitute preferences over three items (where the distribution of value for the third item Normal with mean \$265 and standard deviation \$45), and scaling the number of sellers to \$3,000 from \$2,000 to keep the same number of each item supplied on average. See Table 4. When the values across the three items are positively correlated for a buyer, less items are sold and the market efficiency falls.<sup>31</sup>

As a third experiment with substitutes preferences, we investigate the extent to which the *number* of items for which each buyer has some value can affect the performance of the market. For this, we consider a market with 1,000 buyers and 1,000 sellers and 10 different kinds of items. Buyers possess substitutes preferences on between 1 and 10 items, always with uncorrelated

<sup>31</sup> There being no simple way to define negatively correlated values on three items we just present results for uncorrelated and positively correlated values.

values across items.<sup>32</sup> The value on a bundle of items is defined as

$$v_i(L) = \max_{G_k \in L} v_i(k), \quad (5.1)$$

i.e. these are pure substitutes valuations. Figure 4 illustrates the average market performance of the options-based scheme. As the number of items in which a buyer is interested increases (the “size” of a buyer’s valuation), the additional competition reduces buyer surplus. In addition, efficiency falls because the holdup problem gets worse. However, it is interesting that the number of items sold remains fairly constant at around 53%. In Section 5.4 we will see that the effect of the size of a buyer’s substitutes valuation on efficiency depends on the buyer-to-seller ratio and is significantly improved for higher buy-side competition.

### 5.3 Simulation: Complements Preferences

We now consider buyers with complements valuations among items. While a buyer has a value on each item by itself, in this simulation she is also interested in acquiring both items. The synergy (or lack thereof) of acquiring both items is provided via a Gaussian distributed *multiplicative factor*,  $\beta \in (-1, 1)$ , of the sum of the values of the individual components of the bundle, such that the value for two items,  $v_i(\{A\} \cup \{B\}) = (1 + \beta)(v_i(\{A\}) + v_i(\{B\}))$ , and we define  $\beta \sim N(0, 0.1)$ . We also consider correlation (both negative and positive) across the single-item values on different items. We simulate a market with 5,000 buyers and 2,000 sellers and 120 days, adopting the same set-up as in the substitutes experiments.

Table 5 shows summary statistics for the performance of the market. The average bundle size is the average number of items allocated to buyers, conditioned on winning buyers. This increases from uncorrelated to correlated values across items. This time the correlation seems unimportant, and no matter whether values across items are positively or negatively correlated, we see that around 80% of the items are sold. The number of items sold is relatively high because buyers who hold multiple options can generally exercise both options because of their complements valuations. The number of items sold is not limited as much because of holdup with complements as with substitutes preferences. An individual buyer’s surplus tends to be higher when values are positively correlated because, conditioned on winning at all, she is likely to have higher value on both items and thus

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<sup>32</sup> The specific distributions from which valuations are drawn are as follows:  $v_i(1) \sim N(260, 45)$ ,  $v_i(2) \sim N(240, 10)$ ,  $v_i(3) \sim N(250, 5)$ ,  $v_i(4) \sim N(280, 5)$ ,  $v_i(5) \sim N(260, 20)$ ,  $v_i(6) \sim N(230, 55)$ ,  $v_i(7) \sim N(220, 60)$ ,  $v_i(8) \sim N(245, 5)$ ,  $v_i(9) \sim N(260, 5)$ ,  $v_i(10) \sim N(290, 5)$ .

Buyers' Values	Items Sold (%)	Number of Buyers	Average Bundle Size	Total Value	Buyer Surplus (%)	Sellers Returned (%)
- Correlated	80.0	1,253	1.28	477,388	6.4	31.4
Uncorrelated	80.7	1,139	1.42	487,841	7.5	29.7
+ Correlated	77.3	926	1.67	477,257	9.0	35.1

Table 5

Market performance with 5,000 buyers and 2,000 sellers in a 120 day marketplace. Buyers possess complements preferences over 2 items.

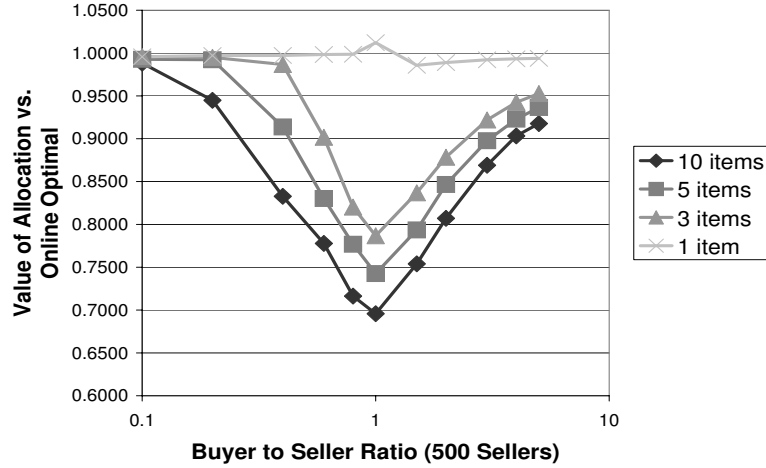


Fig. 5. Efficiency of the options-based scheme at various buyer-to-seller ratios (averaged over 30 runs). Buyers have substitutes preferences on different numbers of items. Number of sellers fixed at 500. Number of days fixed at 120.

higher surplus (while continuing to benefit from price matching). In a simulation in which the number of sellers is increased to 3,000 and buyers have complements valuations on *three* items, but otherwise unchanged, the number of items sold remains at around 80% of supply for both uncorrelated and positively correlated across-item values.

#### 5.4 Simulation: Market Liquidity

For our final study we consider buyers with substitute preferences and uncorrelated values across items and vary the liquidity in the market. For this, we vary the *buyer-to-seller* ratio, which is the ratio of the total number of buyers to sellers in the market, and fix the number of sellers at 500. Figure 5 plots the efficiency (calculated as the ratio of total value of goods allocated in the options-based scheme to a greedy online allocation that approximates the total possible realizable value) against buyer-to-seller

ration.<sup>33</sup> We adopt this online benchmark because we are interested to understand how our solution to the strategic problem (through options, proxies and price-matching) affects performance in relation to a non strategyproof online algorithm. scheme.

What is particularly interesting is that efficiency initially tends to decrease with increasing buyer-to-seller ratio (for all numbers of items other than one) but then increases again.<sup>34</sup> For extremely low buyer-to-seller ratios there is little competition in the market and buyers do not holdup each other too badly and the efficiency is high even with substitutes valuations on many items. As the buyer-to-seller ratio increases efficiency falls as more buyers are blocked because of the holdup problem. At some point, though, efficiency begins to increase again because it becomes less likely that any single buyer will be competitive on *multiple* items. The effect is to separate the market across different types of goods, with each buyer tending to only win on one or two goods and therefore causing only a slight holdup problem and providing higher efficiency. On eBay, for example, we see a buyer-to-seller ratio well above 5:1 in the LCD market and might expect efficiency to remain high even when buyers have large substitutes valuations.

## 6 Discussion: Improving Market Efficiency

The empirical study in the previous section notwithstanding, two factors that limit the market efficiency of the options-based scheme are:

- (1) Proxy agents hold onto options that they will likely not exercise.
- (2) Proxy agents bid their maximum marginal value for options.

Regarding the first point, notice that while proxy agents exercise every option that they hold when items have constant marginal value (e.g., when a buyer wants a single item, or has a linear valuation function), a number of

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<sup>33</sup> The greedy online allocation looks at all buyers and sellers in the market in each day (i.e., considering all auctions on that day) and computes an optimal provisional allocation of goods among the population using a mixed-integer program. For each pair of buyers and sellers that have been matched in the allocation if either one departs at the end of the day then the trade between them is finalized. Otherwise, both buyers and sellers carry over into the next day and continue to be available for provisional allocation.

<sup>34</sup> For the problem instance where buyers are only interested in a single item, efficiency is always very close to 100% for all buyer-to-seller ratios. Note also that while there is a data point above 100% this is not a spurious result because the baseline greedy online heuristic is not guaranteed to be optimal, and it is possible that the options-based scheme can outperform the greedy online heuristic.

options will be returned in general. This occurs quite frequently in our experiments, although a number of these items are ultimately sold upon reposting in a new auction. One simple improvement that can be adopted is to allow a seller who wishes to leave the marketplace but has an issued option to leave (with the good) if the proxy already knows that the option will definitely go unexercised. In so doing, sellers need only remain in the market while there is some possibility that their option will be exercised. However, it is difficult to provide sellers with additional flexibility, for example to allow a seller to offer multiple options for the same unit of a good, without compromising the strategyproofness of the market for buyers. If faced with options that do not provide a *definite* right to exercise and receive a good (for instance if two options are issued on the same unit and both proxies seek to exercise them), then proxy agents would have an incentive to bid for multiple options on the same good.

Regarding the second point, notice that proxy agents may be submitting excessively large values when bidding maximum marginal values. Even if a proxy is already guaranteed utility  $z$  on a bundle of options (based on their current exercise price), the proxy will still go ahead and bid for options on items that could not possibly bring utility of more than  $z$  at *any* exercise price, including zero. The bid price is not adaptive to options, and exercise prices, already secured. We would wish to provide a proxy with a less aggressive bidding strategy and prevent a proxy from acquiring options on items that are currently priced too high to be exercised and especially those that will never be exercised. A natural candidate for a less aggressive bidding strategy – which would still provably acquire all options that could possibly be useful – is to bid the *maximum willingness to pay* given its current allocation of options and the current exercise prices on these options. That is, the proxy should factor in its current state in deciding how much to bid.

For example, suppose that Alice values exactly one piece of fruit (either an apple at \$10, a banana at \$5, or an orange at \$5). If Alice’s proxy already holds an option for an apple with an exercise price of \$8, she might only bid \$3 (instead of \$5) in future auctions for bananas and oranges because securing an option for a banana or an orange at a price above \$3 would only be dominated by the apple option. Similarly, if Alice’s proxy already holds an option for an apple with an exercise price of \$2, she should not bid at all for bananas or oranges, as the maximum surplus possible from acquiring a banana or an orange is guaranteed to be less than the surplus of  $10-2=\$8$  already guaranteed for the apple.

Formalizing, let  $\mathcal{O}^t$  denote the set of options held by a proxy at time  $t$ ,  $\gamma(L) \subseteq G$  denote the goods that correspond to some subset  $L \subseteq \mathcal{O}^t$  of these options, and  $p^t(L) = \sum_{k \in L} p^t(k)$  denote the total exercise price in period  $t$  for this set of options where  $p^t(k)$  is the current exercise price on the option



for good  $G_k$ . Let  $\hat{u}_i^t(L) = \hat{v}_i(\gamma(L)) - p^t(L)$  denote the reported utility of the buyer for options,  $L$ . Let  $L^{t*}$  denote the set of options that maximizes this reported utility. Given this, define the *maximal willingness to pay* for an option on an item  $k$  given the current state of the proxy as:

$$wtp_i^t(k) = \max_S [0, \min[\hat{v}_i(S \cup \{G_k\}) - \hat{u}_i^t(L^{t*}), \hat{v}_i(S \cup \{G_k\}) - \hat{v}_i(S)]], \quad (6.1)$$

where  $\hat{v}_i(S \cup \{G_k\}) - \hat{u}_i^t(L^{t*})$  considers the utility already guaranteed with the current options, and  $\hat{v}_i(S \cup \{G_k\}) - \hat{v}_i(S)$  is the maximal marginal value of good  $G_k$ . This expression calculates the greatest amount a buyer will possibly be willing to spend on an item given the current options held and with uncertainty as to what future items will appear in auctions and about future option prices (assume all  $S \cup \{G_k\}$  are free while the prices on items in  $L^{t*}$  remain the same). However, this scheme cannot be implemented in the proxied architecture without forfeiting truthfulness:

**Example 7** *Both Alice and Bob have substitutes valuations. Alice values either one ton of sand for \$2,000, one ton of stone for \$1,900 and both for \$2,000. Bob values either one ton of sand for \$1,800, one ton of stone for \$1,500 and both for \$1,800. Both buyers have a patience of 2 days. On day one, a sand auction is held, where Alice's proxy bids \$2,000 and Bob's bids \$1,800. Alice's proxy wins an option to purchase sand for \$1,800. On day two, a stone auction is held, where Alice's proxy bids \$1,700 [as she has already obtained a guaranteed \$200 of surplus from winning a sand option, and so reduces her stone bid by this amount], and Bob's bids \$1,500. Alice's proxy wins an option to purchase stone for \$1,500. At the end of the second day, Alice's proxy would exercise the option she holds for stone with an exercise price of \$1,500 to obtain a good valued for \$1,900, and so obtains \$400 in surplus. Now consider what would happen if Alice instead lies, declares that she values only stone, and for \$1,900. On day one, a sand auction is held, where Bob's proxy bids \$1,800. Bob's proxy wins an option to purchase sand for \$0 (because Alice's proxy stays out). On day two, a stone auction is held, where Alice's proxy bids \$1,900, and Bob's bids \$0 [as he has already obtained at least \$1,800 of surplus from winning the sand option, and so is not interested in winning the stone option]. Alice's proxy wins the stone option with an exercise price of \$0, achieving \$1,900 in surplus.*

By misrepresenting her valuation, Alice was able to secure higher surplus by giving more surplus to Bob and therefore reducing the competition that she faced in a future auction. The basic tenet of strategyproof mechanisms requires that the prices faced by a bidder, such as Alice, should be independent of her strategy. This has been compromised because there is now some potential for Alice to influence Bob's bids in the future and in turn the price that she will face. The technical problem is that the value,  $\hat{u}_i^t(L^{t*})$ , in Equation 6.1 is the amount of surplus already guaranteed to

buyer  $i$ , and this now depends on the proxy bids of some other buyer. A slight modification to the option-based scheme, presented in Juda [31], can address this problem in a restricted (“set-valued”) class of valuation domains, namely those in which there are two kinds of goods  $A$  and  $B$ , and each buyer is either indifferent between  $A$  and  $B$  or interested in the bundle  $AB$ . However, we do not know of a remedy to this problem that will allow proxy agents to bid adaptively in more general valuation domains and leave this for future work.

We see a number of additional directions for future work. These include:

- (a) *Allowing buyers to return options to the market early.* We ask whether a scheme can be developed in which a buyer can return an option as soon as her proxy determines that the option will never be exercised given the other options it holds. Such a return would reduce the holdup problem and improve efficiency.
- (b) *Allowing buyers to demand multiple units of a particular item.* This seems quite challenging because a naive solution will leave proxies facing a tradeoff between competing to acquire additional options on a particular good or choosing not to compete, perhaps eliciting greater opportunities for price matching on options that they already hold.
- (c) *Allowing sellers to have values on bundles rather than items.* Scenarios may exist where sellers are interested in selling multiple items in a single lot. The amount of information required to perform book-keeping and track the minimal price possible across auctions is greater than in the current system; in particular, it seems that all bundles in which a bumped buyer is interested would need to be stored.
- (d) *Allowing a buyer’s value to vary with time.* For example, a buyer’s valuation may decrease over time. The difficulty in retaining the dominant-strategy equilibrium property of the market is that a naive solution will leave a proxy with the need to decide if it should delay the exercising of its option, risking the degradation of value but possibly gaining a lower exercise price.
- (e) *Mitigating strategic problems facing sellers.* While our results suggest that sellers can increase their revenue over the eBay protocol when all sellers participate and are straightforward – reporting true arrival and departure information – such behavior may not always be in the best interest of each seller. For example, a seller may do better by delaying her entry into the market if she believes the market is currently “cold” but will soon become “hot” such that there is more competition and higher prices.

(f) *Exploring hierarchical options-based schemes.* The use of options in this paper shifts the sequential auction problem across auctions in a site to a similar problem across different auction sites (e.g. eBay and Yahoo!). It is interesting to consider whether a hierarchical options-based scheme can be developed to help to mitigate the strategic complexity that buyers likely face when bidding across sites.

## 7 Conclusions

We have proposed a novel, options-based method for resolving the strategic difficulties faced by buyers in uncoordinated electronic marketplaces. Our solution to the sequential auction problem is to require that buyers submit bids to *mandatory* bidding proxies which then bid for options on goods and exercise options to maximize reported buyer utility. By allowing proxy agents to match the exercise prices of options to the lowest price that would have been possible through careful timing of the proxy's bid, it becomes a dominant strategy for buyers to report their true value and true temporal constraints to the market. The proxy agents act across multiple auctions and bring simple, truthful bidding into an equilibrium while retaining the dynamic arrivals and departures of open, Internet marketplaces and operating without batching auctions. An empirical analysis that is informed by data collected on eBay for the auctions of LCD monitors suggests that the options-based scheme can provide an improvement in efficiency and revenue over eBay of around 4% and 9% respectively. A series of experiments to examine the effect of a holdup problem that can exist when buyers have general valuations shows that this is mitigated when competition is either very low or high, or when individual buyers have negatively correlated values across items. Efficiency also remains relatively high when buyers have complements valuations, where efficiency is promoted when sellers can re-post goods for sale whose options go unexercised.

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